Optimized Schedules for Airline Routes
Sze-Wei Chang\(^1\) and Paul Schonfeld, F.ASCE\(^2\)

**Abstract:** Increasing flight frequency on airline routes tends to reduce user delay costs but increase airline operating costs. Flight frequency should vary as demand intensity changes at various times of day. Models are presented here for optimizing the departure times of flights on single airline routes with time-dependent demand. An analytic model is developed to find the departure times that minimize average schedule delay per passenger for given flight frequencies and to identify the algebraic relations among the variables. Using an analytic approach, the departure times of flights that minimize airline operating cost and users’ waiting cost are determined. To minimize the objective function, it is found that the headways should be inversely proportional to the square root of demand intensity near the departure times. Computer models for both cost minimization and profit maximization are developed to solve the problems with relaxed assumptions and verify the numerical results obtained analytically.

**DOI:** 10.1061/(ASCE)0733-947X(2004)130:4(412)

**CE Database subject headings:** Airlines; Optimization; Scheduling; Routes; Computer models; Costs.

**Introduction**

It is well recognized in the literature that airline operations must be modeled in considerable detail in order to obtain meaningful airline schedules. Representing all the details leads to ever-increasing model complexity and to prohibitively many variables. According to Etschmaier and Mathaisel (1985), the consensus is that the airline scheduling problem is unsolvable exactly by optimization techniques. It is therefore recognized that the airline scheduling problem is best subdivided into many components and solved through a structured planning process assisted by today’s capable computers, in which all parts of the airline participate.

Airline operators are most concerned with operational decisions, in terms of flight frequency, departure time, aircraft type, and fleet size to serve specific routes at the lowest possible cost. For users, the level of service, in terms of flight frequency and departure time, is most important. In order to reduce their possible waiting time, passengers prefer the highest flight frequency and most convenient departure times. However, those should only be increased up to a specific point that is determined by an airline operator’s economic interests. The optimization of flight frequency and departure times for one route between two cities is considered.

To develop a general case, we consider a single route between two cities and study the operational decisions for this route. The demand intensity over time, the unit costs per aircraft departure, and the user time value are assumed to be known. While we can easily find methods to solve steady demand cases, we are most interested in solving time-varying (i.e., time-dependent) demand cases. An analytic approach will be developed to explore the interrelations among variables. Then, computer models (both cost minimization and profit maximization) for solving such problems will be developed to check the results of the analytic approach. The computer model will have fewer assumptions than the analytic model and provide more accurate results. Ideally, the two approaches should complement each other.

**Airline Scheduling**

The stepwise approach for airline schedule construction that was described by Etschmaier and Mathaisel (1985) provides a conceptual process for airline operational strategy development. Stepwise approaches begin by selecting routes that are to be served and determining the frequency of service on each route. This step is called frequency planning or frequency optimization. The second step determines departure times based on the daily variability of demand. In the third step, departure times are checked for operational feasibility. Based on this stepwise approach, the optimal flight frequency and departure times are solved separately. In this paper, we solve the first two steps jointly, with both analytic and numerical approaches. The third step involves many aspects of the airline operational constraints and will not be included in this paper. Aircraft seating capacity and fleet availability are typical of such constraints. To simplify the development of the analytic models, the following three assumptions are made:

1. A sufficient number of aircraft with unlimited seating capacity is available; therefore, all the passengers can be served with the nearest flight to their preferred departure time.
2. The direct operating cost per flight is constant.
3. Competition effects are negligible on the route.

Thus, our models do not consider how aircraft may be shared with other routes or whether departures during more congested periods are more costly. Newell (1971) developed an analytic model to optimize dispatching times for a public transportation route. Most public transportation systems may have a fixed predetermined schedule, but do not require advance reservations like scheduled airline services.

\(^1\)Assistant Professor, Division of IAD, ROC Minister of National Defense, P.O. Box 90016, Taipei, Taiwan, ROC.

\(^2\)Professor, Dept. of Civil and Environmental Engineering, Univ. of Maryland, College Park, MD 20742. E-mail: pschon@eng.umd.edu (corresponding author).

Note. Discussion open until December 1, 2004. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on August 16, 2000; approved on July 31, 2003. This paper is part of the *Journal of Transportation Engineering*, Vol. 130, No. 4, July 1, 2004. ©ASCE, ISSN 0733-947X/2004/4–412–418/$18.00.
operations. Their demand serving process is that passengers arrive at the station and board the next available departing vehicle. Fig. 1 shows the continuous cumulative demand function with seven departures in a daily operations period for public transportation systems. The shaded area in Fig. 1 shows the total schedule delay, which includes the total time difference between passengers’ preferred departure time and the next departure time. Optimizing the total cost leads to a headway that is inversely proportional to the square root of the demand density.

For a special case in which passengers reserve the latest departure time that occurs earlier than their desired departure time, the shaded area would be on the opposite side of the cumulative demand line, except for the first flight. Assuming a smooth curve the shaded area would be exactly the same, except for the first departure.

A more realistic service process reflecting the situation facing scheduled airlines should be based on minimizing schedule delay, i.e., the difference between the desired and nearest available departure, rather than waiting time to the next departure.

Teodorovic (1983) followed Gagnon’s (1967) assumption that passengers will choose the flight departure time nearest to their desired departure time. Teodorovic (1983) developed models to measure the level of service by minimizing the time difference between actual and desired departure times. In the first model, he demonstrates that the time difference can be approximately expressed as a function of flight frequency only, without regard to the departure times during the day. In the second model and numerical example, he finds the optimal departure times with respect to minimal average schedule delay for a known demand and preassigned frequency on a route between two cities. Schedule delay is defined as the time difference between the desired departure time and the actual departure time.

Minimizing Schedule Delay

We will follow and expand Teodorovic’s (1983) work to evaluate the optimal frequency and departure times for a known time-dependent demand on a route between two cities. In the first part of this paper, a more precise method is developed to find optimal departure times that minimize average schedule delay. In the second part, both an analytic approach and a computer model are used to further evaluate the optimal frequency and departure time.
times that minimize cost and demonstrate the relation between departure headway and unit departure costs.

In Fig. 2, the cumulative demand at a desired departure time is shown and the shaded area represents the total schedule delay. Let \( q(t) = \text{demand intensity (desired departure rate) at time } t; Q(t) = \int q(t) \, dt \) be the cumulative demand at time \( t; t_j = \text{time of the } j\text{th departure}; h(t_j) = \text{headway of the } j\text{th departure time}; W = \text{total schedule delay for all passengers per day}; \) and \( w_j = \text{total schedule delay for all passengers of the } j\text{th departure}. \) The total schedule delay can be formulated as a summation of the total shaded areas:

\[
W = \sum_{j=1}^{y} \left\{ \int_{t_{j-1}+t_j/2}^{t_j} \left[ Q(t) - Q\left(\frac{t_{j-1}+t_j}{2}\right) \right] \, dt + \int_{t_{j-1}+t_j/2}^{t_j+t_{j+1}/2} \left[ Q\left(\frac{t_j+t_{j+1}}{2}\right) - Q(t) \right] \, dt \right\}
\]

(1)

By applying the first-order condition, we minimize the total schedule delay with respect to departure time. Because the schedule delay at departure time \( t_j \) is dependent on the previous departure time \( t_{j-1} \) and the next departure time \( t_{j+1} \), the summation equation can be represented as three departure times, \( t_{j-1} \) (represented as the first term in the following equation), \( t_j \) (second and third terms), and \( t_{j+1} \) (fourth term); that is

\[
\frac{\partial W}{\partial t_j} = \frac{\partial}{\partial t_j} \int_{t_{j-1}}^{t_j} \left[ Q\left(\frac{t_{j-1}+t_j}{2}\right) - Q(t) \right] \, dt + \frac{\partial}{\partial t_j} \int_{t_{j-1}+t_j/2}^{t_j} \left[ Q(t) - Q\left(\frac{t_{j-1}+t_j}{2}\right) \right] \, dt + \frac{\partial}{\partial t_j} \int_{t_{j-1}+t_j/2}^{t_j+t_{j+1}/2} \left[ Q\left(\frac{t_j+t_{j+1}}{2}\right) - Q(t) \right] \, dt + \frac{\partial}{\partial t_j} \int_{t_{j-1}+t_j/2}^{t_j+t_{j+1}/2} \left[ Q(t) - Q\left(\frac{t_j+t_{j+1}}{2}\right) \right] \, dt
\]

\[= 0\]

Solving for \( Q(t_j) \), we find:

\[
Q(t_j) = \frac{1}{2} \left[ Q\left(\frac{t_{j-1}+t_j}{2}\right) + Q\left(\frac{t_j+t_{j+1}}{2}\right) \right]
\]

(2)

This means that the number of passengers whose desired departure time is in the second half of the headway will be equal to the passengers in the first half of the headway. Whatever the headway is, the optimal departure times should be located at the midpoints of two equal sets of passengers. As shown in Fig. 2, the lines d-b and b-e should be exactly equal. Eq. (2) points out the relations among variables which minimize the average schedule delay.

For a daily airline operation with a 24 h demand distribution, Eq. (2) can be directly applied to evaluate the optimal average schedule delay. Because the demand is usually zero at some times during the night, daily demand distributions would normally cover less than 24 h. Teodorovic (1983) used a 16 h demand distribution in his numerical example. Thus, when we consider the passengers in the two tail areas (before the first flight and after the last flight of each day), we have to further define

\[
Q\left(\frac{t_{j-1}+t_j}{2}\right) = Q(t_0) \quad \text{when } j = 1,
\]

\[
Q\left(\frac{t_j+t_{j+1}}{2}\right) = Q(t_{24}) \quad \text{when } j = y.
\]

Table 1. Revised Departure Times versus Teodorovic’s Results

<table>
<thead>
<tr>
<th>Flight</th>
<th>Teodorovic study</th>
<th>Revised results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Departure time</td>
<td>Passengers</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.9215 h</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.0257 h</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3.4867 h</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4.9563 h</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>6.9615 h</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>10.6239 h</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>12.6087 h</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>14.2787 h</td>
</tr>
</tbody>
</table>

Average schedule delay 28.69 min/passenger 27.2781 min/passenger

Minimizing Cost with Respect to Departure Times

Higher service frequency reduces the total schedule delay and increases the level of service for passengers. However, high flight
frequency also increases the airline operator’s cost. In optimizing frequency and departure times, airlines also seek low operating costs.

As mentioned earlier, passengers whose desired departure time is between the two midpoints of adjacent headways will depart on the same flight. The total number of passengers on a flight is equal to the product of the load factor and the aircraft seating capacity; that is

$$Q \left( \frac{t_j + t_{j+1}}{2} \right) - Q \left( \frac{t_{j-1} + t_j}{2} \right) = s(t_j) \ell(t_j)$$  (4)

where load factor $0 \leq \ell(t_j) \leq 1$.

By solving Eqs. (2) and (4) jointly, we can obtain

$$s(t_j) \ell(t_j) = 2Q(t_j) - 2Q \left( \frac{t_{j-1} + t_j}{2} \right) + 2 \int_{(t_{j-1} + t_j)/2}^{t_j} q(t) dt$$  (5)

For a smooth curve $Q(t)$ and frequent departures, we assume that the lines a-b and b-c in Fig. 2 are nearly straight and that $q(t)$ at $a$ and $b$ are the slopes for these lines. As a result of Eq. (2), lines d-b and b-e are exactly equal. By solving graphically and using the results of Eq. (2), we find that

$$h(t_j) = \frac{t_j - t_{j-1}}{2}$$

where the headway $h(t_j) = t_j - t_{j-1}$ is

$$h(t_{j+1}) = h(t_j) \sqrt{1 + \frac{t_j - t_{j-1}}{2}} \quad q(t_j)$$  (6)

Because demand density, $q(t)$, is given with Eq. (6), we can iteratively determine each of the next headways numerically, after the previous headway has been determined. Eq. (6) is derived from Eq. (2) under the assumption of a smooth curve $Q(t)$ and frequent departures.

As mentioned earlier, the schedule delay for passengers in the $j$th departure is represented as the shaded area in Fig. 2. The schedule delay for passengers in the $j$th departure is close to the sum of the areas of two triangles, a-b-e and b-d-c; that is

$$w_j = \frac{1}{2} \left[ Q(t_j) - Q \left( \frac{t_{j-1} + t_j}{2} \right) \right] h(t_j) + \frac{1}{2} \left[ Q \left( \frac{t_j + t_{j+1}}{2} \right) - Q(t_j) \right] h(t_{j+1})$$  (7)

Substituting Eqs. (5) and (6) into Eq. (7), we have

$$w_j = \frac{1}{8} \int_{(t_{j-1} + t_j)/2}^{(t_j + t_{j+1})/2} h(t_j) \left[ 1 + q(t_j) \right] q(t_j) dt$$  (8)

An approximate expression for the total schedule delay is

$$W = \frac{1}{8} \int_{t_0}^{T} h(t_j) \left[ 1 + q(t_j) \right] q(t_j) dt$$  (9)

The total schedule delay can be minimized by decreasing the headway as much as possible. Eq. (7) indicates that there is no delay at all when the headway approaches zero. Realistically, short headways result in more flights per day, and higher operating costs. On the other hand, even if an airline provides sufficient flight frequency, improper departure times also increase passenger schedule delay cost. Thus, the service frequency and departure times are the decision variables for minimizing the total cost. Newell (1971) defined the dispatch frequency during the time $t_0$ to $T$ as

$$y = \int_{t_0}^{T} h(t_j)^{-1} dt$$  (10)

Earlier, in Eq. (1), the waiting time, $W$, was minimized for a given number of flights. Adding the cost of flights (which is the direct operating cost per flight, $a$, multiplied by the number of flights per day, $y$) to the objective function, we obtain an expanded total cost objective function, in which $c$ is the time value of passenger waiting time:

$$C = Wc + ay = c \int_{t_0}^{T} h(t_j) \left[ 1 + q \left( \frac{t_j + t_{j+1}}{2} \right) \right] q(t_j) q(t) dt$$

By minimizing the objective function with respect to headway, we can obtain

$$h_j^* = t_j - t_{j-1} = \frac{8a/c}{q(t_j) + q(t_j + t_{j-1}/2)}^{1/2}$$  (11)

The result of headway optimization for airlines is quite different from headway optimization for public transportation systems, as done by Newell (1971). Given the different service process of public transportation systems as compared to scheduled airlines, Newell’s (1971) optimal headways are inversely proportional to the square root of the current arrival rate (demand density). However, in our scheduled airline case, the headways minimizing the total schedule delay depend on two adjacent departure rates. Consequently, the headways are inversely proportional to the square root of the sum of the departure rates in the current and previous periods. Given any stating value of $h_1$, Eq. (11) is used to uniquely determine all the remaining headways $h_2, h_3, \ldots, h_y$. The different values of $h_1$ determine many different headway sets, of which one should be optimal. Thus, for any given $h_1$, the conditionally optimized values $h_2$ through $h_y$, as well as the total values of the objective function, $C$, are uniquely determined with Eq. (11). In this way, a multidimensional search for the optimal set of headways can be reduced to a one-dimensional search for the $h_1$ value that minimizes the objective function, $C$.

Eq. (11) also shows that headways increase with the square root of unit cost per flight and decrease inversely with the square root of the users’ time value. Consequently, the flight frequency decreases with unit cost per flight and increases with passenger time value. Again, Eq. (3) also applies to Eq. (11) to cover the two tails of the demand distribution.
To verify the result, we use the following numerical example, in which we adopt Teodorovic’s (1983) demand density (desired departure rate) and cumulative demand function. Those are the results of a poll of passengers flying from Belgrade to Zagreb, Yugoslavia, in 1980:

\[
q(t) = -0.0802t^4 + 2.5827t^3 - 25.9952t^2 + 79.9252t + 48.2199
\]  

\[
Q(t) = \int q(t)dt = -0.016t^5 + 0.6457t^4 - 8.6651t^3 + 39.9626t^2
+ 48.2199t 
\]  

(12)

The demand density and cumulative demand function have two typical peak hours in a daily operation, as shown in Fig. 4. Let \(a = 1,000 \, \text{$/flight and } c = 10 \, \text{$/h}. By applying Eqs. (11) and (3) as well as the trial value of \(h_1\), we can calculate headways sequentially. Table 2 shows those results.

Eq. (11) is solved by determining a trial value for first headway, \(h_1\), and then solving sequentially for the following headways, \(h_2...h_y\). These form a headway set, shown in the second row in Table 2. Many headway sets are produced as the value of \(h_1\) changes, of which one should be optimal. When we review the first and last flight, we see that the first flight has a full load of passengers and the last flight has excess capacity. To improve that, we shift all flights earlier to seek the optimal first headway. Table 3 shows the optimal headway set obtained through the analytical approach. The improvement in total schedule delay and total cost is evident. Actually, the proposed computer model in the next section is capable of finding the first optimal headway by itself. It can be used to verify the optimal headway set of Table 3 and provide more accurate results.

### Computer Model

The analytic model developed herein has at least three assumptions that limit its applicability. The assumption that the seat capacity is unlimited is usually unrealistic, but it simplifies the problem. Although seat capacity constraints can be included in the model, many cost functions for acquiring and operating different types of aircraft must then be involved in the objective function. In order to verify the results of the analytic approach, we prefer to retain assumption 1. With computer assistance, the assumptions of a smooth curve \(Q(t)\) and frequent departures can now be relaxed. The resulting formulation, which represents the characteristics of the problem, is

\[
\begin{align*}
\text{Minimize} & \quad c \left( w_1 + \sum_{j=2}^{y-1} w_j + w_y \right) + ay \\
\text{Subject to} & \quad t_j = t_{j-1} + h_j \quad (21) \quad t_j \leq 16 \\
& \quad t_j \geq 0 
\end{align*}
\]  

(17)

While Eq. (15) represents the total schedule delay by which the passengers delay or advance their preferred departure to catch each flight, Eqs. (14) and (16) include the total schedule delay at the tails of demand distribution for the first and last flights. Eq. (17) represents the total cost objective function. Eq. (18) ensures that the departure time of the next flight is never earlier than the previous flight. Eq. (19) sets initial values for scheduling time and headway. Constraint 20 requires that all flights be scheduled
within the 16 h operating period in each day, while constraint 21 ensures nonnegative departure times and headways.

With constraint functions, bounds, and initial values, the model is used to optimize the objective function. The model searches the possible headway combination sets and evaluates them with respect to optimal objective function. While solving the previous example, the model was running about 675 iterations in 11 min on a 266 MHz processor to identify the optimal headway set. The optimal frequency is six flights per day and the total cost was slightly improved again from the last example. The first part of Table 4 shows the total cost comparison among different daily frequencies. The computer model results are more precise than those of the analytic model. The total cost of the optimal headway set in the analytic model ($12,640) was improved to $12,259 in the computer model. Table 4 also shows the computer model results for the new optimized headways and departure times. With known demand intensity and unit costs, the results confirm the implications of Eq. (11). The slight difference in results between two models is due to the relaxation of the assumptions of a smooth curve $Q(t)$ and frequent departures.

Eq. (2) is the baseline for deriving the result of Eq. (11) in the analytic model, but it is not built into the computer model. When examining the passenger load in Table 4, it is notable that the relation in Eq. (2) is met in the optimality condition. This relation also applies to the two tails of the demand distribution. Of course, further evaluation of the data in Table 4 can more precisely demonstrate the same interrelations as Eqs. (2) and (11).

Fig. 5 plots the computer model output. As we can see, the cumulative demand line crosses over the midpoint of the passenger load for every departure flight and headway decreases as demand density increases.

### Maximizing Profit with Respect to Flight Frequency and Departure Times

In order to reduce their possible waiting time, passengers seek frequent, convenient departure times and seat availability. Of course, these passenger preferences conflict with an airline’s economic considerations. When improving service quality, the airline may increase operating cost and hence sacrifice its profitability. On the other hand, when decreasing the service quality, an airline may reduce its attraction to passengers and consequently its potential market share and profitability. Because service quality to attract passengers is an essential factor in the scheduling process, it must be balanced against the economic considerations.

In this section, the objective function is to maximize the profit, which is total revenue minus operating costs for a single route market. The total revenue is the product of total passengers and ticket price. For a given ticket price ($p$), the number of passengers ($n$) is a function of potential demand ($m$), schedule delay ($W$), and an adjustment factor ($a$). When a deviation from passenger’s satisfaction in schedule delay occurs, passengers turn to other airlines or other transportation modes and the number of passengers on board is below the potential demand.

The decision variables are flight frequency and departure times. The objective can be furthered both by a decrease in schedule delay, yielding more passengers, or by a decrease in operating cost with fewer flights per day. Total schedule delay, $W$, is the sum of schedule delay for all flights [Eqs. (14)–(16)]:

$$W = w_1 + \sum_{j=2}^{y-1} w_j + w_y$$  \hspace{1cm} (22)

$$n = (1 - a W) m$$  \hspace{1cm} (23)

Maximize $pn - ay$  \hspace{1cm} (24)

Subject to $t_j = t_{j-1} + h_j$  \hspace{1cm} (25)

$t_1$ and $h_1 = 0$  \hspace{1cm} (26)

$t_j \leq 16$  \hspace{1cm} (27)

$t_j$ and $h_j \geq 0 \hspace{1cm} \forall j = 1, \ldots, y$ \hspace{1cm} (28)

With constraint functions, bounds, and initial values, the profit is maximized by the trade-off between quality and operating costs. The model searches the possible headway combination sets (flight frequency and departure times) and evaluates them with respect to the objective function.

The first part of Table 5 shows the total profit comparison among different daily frequencies. The result shows that four flights per day is optimal based on the trade-off between demand loss (service quality) and operating costs. The second part of Table 5 shows the optimal departure times and passenger load. It also is notable that the relation in Eq. (2) is met in the optimality condition.

### Table 5. Profit Maximization Model Results

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Total profit</th>
<th>Schedule delay</th>
<th>Actual demand</th>
<th>Optimal $y=6$</th>
<th>$h(t_j)$</th>
<th>Departure time</th>
<th>Passengers/flight</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y=8$</td>
<td>10,978</td>
<td>477.1</td>
<td>949</td>
<td>$t_1$</td>
<td>1.811</td>
<td>1.811</td>
<td>284</td>
</tr>
<tr>
<td>$y=7$</td>
<td>11,732</td>
<td>535.9</td>
<td>937</td>
<td>$t_2$</td>
<td>2.865</td>
<td>4.676</td>
<td>229</td>
</tr>
<tr>
<td>$y=6$</td>
<td>12,354</td>
<td>625.9</td>
<td>918</td>
<td>$t_3$</td>
<td>6.502</td>
<td>11.178</td>
<td>151</td>
</tr>
<tr>
<td>$y=5$</td>
<td>12,906</td>
<td>732.7</td>
<td>895</td>
<td>$t_4$</td>
<td>2.809</td>
<td>13.987</td>
<td>195</td>
</tr>
<tr>
<td>$y=4$</td>
<td>13,189</td>
<td>903.5</td>
<td>859</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y=3$</td>
<td>12,479</td>
<td>1,311.0</td>
<td>773</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

JOURNAL OF TRANSPORTATION ENGINEERING © ASCE / JULY/AUGUST 2004 / 417
Conclusions

The analytic model for minimizing schedule delay provides a simple relation in Eq. (2) for optimizing departure times. Eq. (11) provides a simple and fairly precise relation among optimal frequency, headway set, unit costs, and demand density for minimizing total cost. Both of them improve on previous studies. They offer very useful guides to cost minimization for airlines serving specific routes.

The computer models have fewer assumptions and more precise results than the analytic model. The computer models can also be integrated with other components of the larger airline scheduling process.

The profit maximization model provides useful guidance to airline managers regarding service quality, frequency, and departure time decisions. Decisions about departure times will affect costs, service, and profits even more when these models are incorporated in complete airline scheduling systems for multiroute networks.

Notation

The following symbols are used in this paper:

- \( a \) = direct operating cost per flight ($/flight);
- \( C \) = total cost ($/day);
- \( c \) = time value per schedule delay hour per passengers ($/passenger hour);
- \( h(t) \) = headway at time \( t \) (hours);
- \( l \) = load factor (%);
- \( m \) = potential demand (passengers);
- \( n \) = actual demand (passengers);
- \( p \) = ticket price ($/passenger hour);
- \( Q(t) \) = cumulative demand at time \( t \) (passengers);
- \( q(t) \) = demand density at time \( t \) (passengers/hour);
- \( s(t) \) = aircraft capacity at time \( t \) (seat/aircraft);
- \( t_j \) = time of \( j \)th departure;
- \( W \) = total schedule delay for all passengers per day (passenger hours);
- \( w_j \) = total schedule delay for all passengers of \( j \)th departure (passenger hours);
- \( y \) = service frequency (flights per day); and
- \( \alpha \) = adjusting factor for waiting cost (0.002/wating time).

References