Itinerary-Based Airline Fleet Assignment

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We consider the airline fleet assignment problem involving the profit maximizing assignment of aircraft types to flight legs. Although several basic formulations have been proposed, important network considerations are insufficiently treated in these formulations and the resulting solutions are often suboptimal. We propose a new formulation and solution approach that captures network effects and generates superior solutions. We quantify the benefits of our proposed approach in a case study using data from a major United States airline.

An airline is faced with a number of daunting problems in scheduling its aircraft to meet customer demands. Once an airline decides when and where to fly (i.e., develops a flight schedule), the next crucial decision is determining the type of aircraft, or fleet, that should be used on each of these flight legs. This is referred to as the fleet assignment problem. The objective of the fleet assignment problem is to assign fleet types to flight legs, subject to an available number of aircraft and conservation of aircraft flow requirements, such that the fleeting contribution is maximized. In most basic fleet assignment models, fleeting contribution is defined as unconstrained revenue less assignment cost. Unconstrained revenue of a flight leg, a constant, is the maximum attainable revenue for that particular flight regardless of assigned capacity. Assignment cost, a function of the assigned fleet type, includes the flight operating cost, passenger carrying related cost and spill cost. (§2.2 provides a detailed discussion of these components.) Spill cost on a flight is the revenue lost when the assigned aircraft for that flight cannot accommodate every passenger. The result is that either the airline spills some passengers to other flights in its own network (in which case these passengers are recaptured by the airline), or they are spilled to other airlines. An alternative, equivalent objective function for the basic fleet assignment model, therefore, is to minimize assignment cost.

There are several shortcomings of basic fleet assignment models:

1. In basic fleet assignment models, spill and recapture are ignored or modeled only approximately. That is, estimates for spill cost are often obtained for a flight leg by assuming that capacity is unconstrained on every flight leg, except the one for which the estimate is being made. Estimates of recaptured revenue are achieved without knowledge of capacity or passenger flow on the flight network.

2. Most fleet assignment models consider only aggregate demand and average fares for different fare classes. This can significantly impact spill cost estimates, particularly because most United States airlines employ sophisticated revenue management systems to manage inventory and deliberately spill certain segments of passengers in order to protect seats for higher value passengers. Without accurate representation of demands and fares for different fare classes, the accuracy of estimated spills and spill costs in fleet assignment models can be seriously compromised.
(3) Most fleet assignment models assume that demand is static, even though it varies by day-of-week and season.

Contributions
Our contributions in this paper are:

(1) We develop a new assignment model, an Itinerary-Based Fleet Assignment Model, that is capable of capturing network effects and more accurately estimating spill and recapture of passengers, an improvement over existing models.

(2) We devise a solution algorithm for our itinerary-based fleet assignment model and show its performance on several networks of different sizes, including full-scale networks of a large United States airline.

(3) Through case studies using real airline data, we quantify the individual and joint benefits of more accurately modeling spill and recapture.

(4) We provide a set of validating experiments showing that our model is capable of producing better fleeting decisions for a major United States carrier.

Outline
In §1, we illustrate the importance of considering network effects in assigning fleets. In §2, we review two relevant models: the Passenger Mix Model (PMM) and the basic Fleet Assignment Model (FAM). The Itinerary-Based Fleet Assignment Model (IFAM) and solution approach is presented in §3. In §4, we present a case study performed on two full-scale data sets (provided by a major United States airline) comparing the solution quality using a basic fleet assignment model and our itinerary-based fleet assignment model. In §5, we provide some experiments evaluating the sensitivity of our approach to its underlying demand and recapture assumptions.

Table 1  Demand Data

<table>
<thead>
<tr>
<th>Market</th>
<th>Itinerary (Sequence of Flights)</th>
<th>Number of Passengers</th>
<th>Average Fare</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-Y</td>
<td>1</td>
<td>75</td>
<td>$200</td>
</tr>
<tr>
<td>Y-Z</td>
<td>2</td>
<td>150</td>
<td>$225</td>
</tr>
<tr>
<td>X-Z</td>
<td>1–2</td>
<td>75</td>
<td>$300</td>
</tr>
</tbody>
</table>

Table 2  Seating Capacity

<table>
<thead>
<tr>
<th>Fleet Type</th>
<th>Number of Seats</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>100</td>
</tr>
<tr>
<td>B</td>
<td>200</td>
</tr>
</tbody>
</table>

1. Network Effects
To illustrate the effects of flight leg interdependencies (network effects) on spill and recapture, consider the example detailed in Tables 1, 2, and 3. Table 1 shows demand and fare data in three markets X-Y, Y-Z, and X-Z (connecting through Y). Table 2 shows seating capacity of each aircraft type. Table 3 shows operating costs for each possible fleet to flight assignment. Note that for simplicity, we ignore passenger related costs for the moment. Table 4 lists all possible fleeting combinations for this small example, along with their associated operating costs.

The unconstrained revenue is $75($200) + 150($225) + 75($300) = $71,250. For the following analysis, we assume that the airline has full discretion in determining which passengers it wishes to accommodate. If we choose fleeting I, then each flight leg has a capacity of 100 seats. The demand for flights 1 and 2 is 150 and 225 passengers, respectively. Therefore, we must spill 50 of the passengers who desire travel on flight 1 and 125 passengers who desire travel on flight 2. Since the fare for the X-Z itinerary is less than the sum of the two local itineraries, the revenue maximizing strategy is to first spill 50 passengers on the X-Z itinerary ($15,000). The remaining demand for flight 1 no longer exceeds capacity. Since the local fare for flight 2 is less than the fare for the X-Z itinerary, we spill 75 passengers from the Y-Z itinerary ($16,875). Therefore, the minimum spill costs for fleeting I is $15,000 + $16,875 = $31,875. The spill costs for each fleeting are shown in Table 5.

By definition, then, fleeting contribution for fleeting I is $71,250 − ($30,000 + $31,875) = $9,375. Analyzing
all other fleetings similarly, we see in Table 5 that the optimal fleeting for this tiny example is fleeting I. Consider now the case when we treat each flight leg independently and minimize its own spill cost, independent of the effects on other flights in the network. This is indeed the underlying strategy employed in generating objective function coefficients for basic fleet assignment models. The spill minimizing strategy in this case for each flight is to greedily spill passengers in order of increasing fare until the number of passengers exactly equals the assignment capacity for that flight. In this example, we always spill local passengers in favor of keeping the higher fare connecting passengers. For fleeting I, we would spill 50 X-Y passengers at a fare of $200 and 125 Y-Z passengers at a fare of $225. The resulting spill costs and contribution for each fleeting are in Table 6.

If we use the greedy model, we are indifferent to either fleeting II or IV. The reason for the difference in fleetings between this greedy heuristic and the network approach above is that the greedy model does not capture flight interdependencies or network effects. The best set of passengers to spill from one flight leg is a function of the demands and assigned capacity on other flight legs. The myopic solution can be improved by taking a network-wide view of the problem and spilling connecting passengers and accommodating local passengers.

In this small example, it is possible to enumerate possible fleeting combinations and compute the minimum spill costs accordingly. However, with a network of hundreds or thousands of flight legs, enumeration is computationally expensive, if not entirely impossible. Thus, in this paper, we describe a new modeling and algorithmic approach for fleet assignment that models spill and recapture as a function of assigned capacity across an entire airline network, and not just a single flight leg.

2. Review of Fleet Assignment and Passenger Mix Models

2.1. Terminologies and Notations

2.1.1. Terminologies. To facilitate the description of the passenger mix and fleet assignment models, we define the following terms. A flight leg is a non-stop trip of an aircraft from an origin airport to a destination airport (one take-off and one landing). A market is an ordered origin-destination airport pair, in which passengers wish to fly. Boston Logan International (BOS) to Chicago O’Hare (ORD), for example, is a distinct market from ORD-BOS, which is an opposite market. An itinerary in a particular market consists of a specific sequence of scheduled flight legs, in which the first leg originates from the origin airport at a particular time and the final leg terminates at the final destination airport at a later time. We model a round-trip itinerary as two distinct trips in two opposite markets. A Boston–Chicago round-trip is represented as a passenger in the BOS-ORD market and a passenger in the ORD-BOS market. A market may have numerous itineraries. The itinerary comprised of flight 789 from BOS to ORD and flight 276 from ORD to Los Angeles International (LAX) is distinct from the itinerary comprised of flight 792 from BOS to ORD and flight 275 from ORD to LAX, of which both are in the BOS-LAX market, but scheduled at different times.
Often times the modifier *unconstrained* is used to denote that the quantity of interest is measured or computed without taking into account any capacity restriction. For example, *unconstrained demand* of a market is the total demand in a market (or request for air travel) *as experienced by the airline of interest* in a market regardless of flights or seats offered. This should be distinguished from *constrained demand*, which is the total demand accommodated by the airlines, subject to available aircraft capacity. Similarly, *unconstrained revenue* is the maximum revenue attainable by the airline independent of capacity offered; while *constrained revenue* is the achievable revenue subject to capacity constraints.

We assume our schedule is daily, that is, the schedule repeats itself everyday. Thus, the end of the day is connected to the beginning of the day by a wrap-around arc. The extension to weekly schedule is straightforward. Airlines, however, prefer solving daily schedule because they can maintain fleeting consistency throughout the week and thus minimize operational complications.

In order to count the number of aircraft in the network, we take a snapshot of the network at some point in time and count the number of aircraft both in the air and on the ground at stations. The exact time that we take the snapshot, defined as the count time, does not matter because conservation of aircraft flow is required throughout the network.

### 2.1.2. Notations.

**Sets**
- $P$: the set of itineraries in a market indexed by $p$ or $r$.
- $A$: the set of airports, or stations, indexed by $a$.
- $L$: the set of flight legs in the flight schedule indexed by $l$.
- $K$: the set of different fleet types indexed by $k$.
- $T$: the sorted set of all event (departure or availability) times at all airports, indexed by $t$. The event at time $t$ occurs before the event at time $t_{i+1}$. Suppose $|T| = m$; therefore $t_1$ is the time associated with the first event after the count time and $t_m$ is the time associated with the last event before the next count time.
- $N$: the set of nodes in the timeline network indexed by $\{k, o, t\}$.

### Decision Variables

- $CL(k)$: the set of flight legs that pass the count time when flown by fleet type $k$.
- $I(k, o, t)$: the set of inbound flight legs to node $\{k, o, t\}$.
- $O(k, o, t)$: the set of outbound flight legs from node $\{k, o, t\}$.

**Decision Variables**

- $t^+_i$: the number of passengers requesting itinerary $p$ but are redirected by the model to itinerary $r$.

**Parameters/Data**

- $CAP_i$: the number of seats available on flight leg $i$ (assuming fleeted schedule).
- $SEATTS_i$: the number of seats available on aircraft of fleet type $k$.
- $N_k$: the number of aircraft in fleet type $k$, $\forall k \in K$.
- $D_p$: the unconstrained demand for itinerary $p$, i.e., the number of passengers requesting itinerary $p$.
- $\text{fare}_p$: the fare for itinerary $p$.
- $\text{fare}_r$: the carrying cost adjusted fare for itinerary $p$.
- $\tilde{b}_i^p$: recapture rate from $p$ to $r$; the fraction of passengers spilled from itinerary $p$ that the airline succeeds in redirecting to itinerary $r$.

$$\delta^p_i := \begin{cases} 
1 & \text{if itinerary } p \in P \text{ includes flight leg } i \in N; \\
0 & \text{otherwise.}
\end{cases}$$

### 2.2. The Basic Fleet Assignment Model

The kernel of most Fleet Assignment Models (FAM), referred to as the basic fleet assignment model, can be described as:

- maximize: fleeting contribution
  (or minimize: assignment cost)
subject to: \[ \text{all flights flown by exactly one aircraft type, aircraft flow balance, and only the number of available aircraft are used,} \]
or mathematically as:

\[
\min \sum_{i \in L} \sum_{k \in K} C_{k,i} f_{k,i} \tag{1}
\]

subject to:

\[
\sum_{k \in K} f_{k,i} = 1, \quad \forall i \in L, \tag{2}
\]

\[
y_{k,o,t} + \sum_{i \in I(k,o,t)} f_{k,i} - y_{k,o,t}^+ - \sum_{i \in O(k,o,t)} f_{k,i} = 0, \quad \forall k, o, t, \tag{3}
\]

\[
\sum_{i \in A} y_{k,o,t} + \sum_{i \in I_i(k)} f_{k,i} \leq N_k, \quad \forall k \in K, \tag{4}
\]

\[
f_{k,i} \in \{0, 1\}, \quad \forall k \in K, \forall i \in L, \tag{5}
\]

\[
y_{k,o,t} \geq 0, \quad \forall k, o, t. \tag{6}
\]

The basis for several fleet assignment models currently used by several airlines in the industry is the model proposed by Hane et al. (1995). Constraints (2) are cover constraints ensuring that each flight is covered once and only once by a fleet type. Constraints (3) are conservation of flow constraints ensuring aircraft balance, that is, aircraft going into a station at a particular time must leave that station at some later time. Constraints (4) are count constraints ensuring that only the available number of aircraft of each type are used in the assignment. The objective function coefficient \( C_{k,i} \) is the summation of the following components:

(1) Operating costs: these are flight and fleet specific costs derived specifically from the operation of the flight with a given fleet. These costs, including the minimum amount of fuel, gate rental, and take-off and landing costs, are independent of the number of passengers on board.

(2) Carrying costs: these are the costs dependent on the number of passengers flown, including, but not limited to, the costs of extra fuel, baggage handling, reservation systems processing, and meals. Because the number of passengers on a given flight is a function of the capacity assigned to that flight and to other flights (due to network effects), it is impossible to compute for that flight a single value representing the total carrying cost associated with an assigned fleet type. Hence, in basic FAM, estimates of carrying costs by flight leg for each assigned fleet cannot be exact.

(3) Spill costs: given a fleet assignment, this is the sum over all itineraries, of the revenue spilled from the itineraries due to insufficient capacity. In basic FAM models, this value cannot be captured exactly for two reasons:

(a) Spill costs in FAM models are estimated for each flight leg and each possible fleet assignment to that leg. As illustrated earlier, however, it is impossible to decide which passengers to spill from a particular leg \( i \) unless the capacities assigned to other flight legs (such as those contained in itineraries including \( i \) ) are also known.

(b) Even if the number of passengers of each itinerary to spill from a flight leg were known, it would not be possible to assign accurately the spill costs to individual flights, as required in the basic FAM model and guaranteed that the solution to the FAM model would indeed be optimal.

(4) Recaptured revenue: this is the portion of the spill costs that are recovered by transporting passengers on itineraries other that their desired itineraries. Obviously, if spill is only approximated as in basic FAM models, then recapture is at best approximate. In some basic FAM models, recapture is approximated as some fraction of the (approximated) spill, independent of whether or not capacity exists to transport these passengers.


**Spill Cost Estimation in Basic FAM**

We now detail the spill cost estimation process in basic FAM and indicate its major source of errors.

**Step 1. Fare Allocation: From Itinerary to Flight Leg.**

In this step, itinerary-based passenger fares are allocated to flight legs. For a direct itinerary, fare allocation is straightforward because there is only one flight
leg. For itineraries containing more than one flight leg, several “heuristic” schemes are possible. Two allocation schemes widely used in the industry are (a) full fare allocation; and (b) mileage-based prorated fare allocation. In the former, each leg in the itinerary is assigned the full fare of the itinerary; while in the latter, each leg in the itinerary is assigned a fraction of the total fare, proportional to the ratio of the leg’s mileage to total mileage in the itinerary.

Step 2. Spill Estimation. There are two major approaches to spill estimation once fares have been allocated.

Deterministic: Given the fare allocation of Step 1, spill is determined for each flight leg based on its capacity, independent of other flight legs. The most common spill estimation process considers a flight leg \(i\) and the unconstrained demand (passengers by itinerary) for \(i\). It begins by listing the passengers in order of decreasing revenue contribution, and then offering seats to those on the list, in order, until all passengers are processed or capacity is fully utilized. If the capacity is sufficient to carry all passengers, no spill occurs and spill cost for leg \(i\) is zero. If, on the other hand, demand exceeds capacity, lower ranked passengers are spilled and the total revenue of these spilled passengers is the estimated spill cost for flight leg \(i\).

Probabilistic: The estimated spill cost of assigning fleet type \(k\) to leg \(i\) is computed as the product of an average spill fare, \(SF_k\), and expected number of spilled passengers, \(E[t_{ki}]\). The expected number of spilled passengers, \(E[t_{ki}]\), is estimated for flight leg \(i\) with assigned fleet type \(k\), assuming that the flight leg level demand distribution is Gaussian. The standard parameters for the Gaussian distribution used in this context are: expected demand \(Q_i\), average number of passengers traveling on flight leg \(i\), and standard deviation \(K\times Q_i\), where \(K\) is between 0.2 and 0.5. Alternatively, another popular estimate for the standard deviation is \(Z\sqrt{Q_i}\), where \(Z\) is between 1.0 and 2.5. Details of this process can be found in Kniker (1998).

Clearly, the basic FAM spill estimation step above is inexact for it does not allow for network interdependency. Kniker and Barnhart (1998) experiment with several fare allocation schemes and show that different allocation schemes generate different fleet assignment decisions, different contributions, and that no one scheme consistently outperforms the others.

2.3. Passenger Mix Model

The Passenger Mix Model (PMM) proposed by Kniker et al. (2001) takes a fleeted schedule (that is, each flight leg is assigned one fleet type), and unconstrained itinerary demand as input and finds the flow of passengers over this schedule that maximizes fleeting contribution, or equivalently minimizes assignment cost. Because the schedule is fleeted, flight operating costs are fixed and only passenger carrying and spill costs are minimized. The objective of the model, then, is to identify the best mix of passengers from each itinerary on each flight leg. The solution algorithm spills passengers when necessary on less profitable itineraries in order to secure the seats for the passengers on more profitable itineraries. Hence, the Passenger Mix Problem is:

\[
\text{Given a fleeted flight schedule and the unconstrained itinerary demands, find the carrying plus spill cost minimizing flow of passengers over the network, such that (1) the total number of passengers on each flight does not exceed the capacity of the flight, and (2) the total number of passengers on each itinerary does not exceed the unconstrained demand of that itinerary.}
\]

To illustrate this, consider the example of Tables 7 and 8. Note that in this example (and in PMM) itinerary fare values are adjusted to reflect carrying costs per passenger. Using a greedy algorithm (booking higher fare passengers first) 75 BOS-ORD passengers are booked on flight A, 80 ORD-DEN passengers are booked on flight B, and 40 seats are booked on both flights for BOS-DEN passengers. This algorithm yields a revenue of $33,250. Alternatively, we could book all BOS-DEN passengers first and then assign the remaining seats on both flights to BOS-ORD and ORD-DEN passengers. This approach yields

<table>
<thead>
<tr>
<th>Flight</th>
<th>Origin</th>
<th>Destination</th>
<th>Departure</th>
<th>Arrival</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>BOS</td>
<td>ORD</td>
<td>9:45 am</td>
<td>11:23 am</td>
<td>120</td>
</tr>
<tr>
<td>B</td>
<td>ORD</td>
<td>DEN</td>
<td>12:00 pm</td>
<td>1:31 pm</td>
<td>120</td>
</tr>
</tbody>
</table>

Table 7 PMM Model Example: Flight Schedule
an increased revenue of $33,500. The maximum revenue, however, is $33,750 with 75 BOS-ORD passengers, 75 ORD-DEN passengers, and 45 BOS-ORD passengers assigned. From this tiny example, we can see that the mix of passengers on flights can affect revenues substantially.

To formulate the passenger mix problem, we denote \( \tilde{\text{fare}}_r \) as the adjusted fare for itinerary \( r \), i.e., fare less carrying cost for itinerary \( r \). We let \( t^r_p \) represent the number of passengers spilled from itinerary \( p \) and redirected to itinerary \( r \). In this formulation, we explicitly model recapture phenomenon using recapture rates, \( b^r_p \) for each \( r \in R \), and \( p \in P \), denoting the fraction of passengers spilled from itinerary \( p \) whom the airline succeeds in redirecting to itinerary \( r \). The product of \( t^r_p \) and \( b^r_p \) is therefore the number of passengers recaptured from itinerary \( p \) onto itinerary \( r \). We denote \( t^r_p \) as spill from itinerary \( p \) to a null itinerary and assign its associated recapture rate, \( b^r_p \), a value of 1.0. Passengers spilled from itinerary \( p \) onto a null itinerary are not recaptured on any other itinerary of the airline and are lost to the airline. The mathematical formulation for PMM is:

\[
\begin{align*}
\text{Min} & \quad \sum_{p \in P} \sum_{r \in R} (\tilde{\text{fare}}_r - b^r_p \tilde{\text{fare}}_p) t^r_p \\
\text{subject to:} & \\
\sum_{p \in P} \sum_{r \in R} \delta^r_i t^r_p - \sum_{r \in R} \delta^r_i b^r_p t^r_p & \geq Q_i - \text{CAP}_i, \quad \forall i \in L, \quad (8) \\
\sum_{r \in R} t^r_p & \leq D_p, \quad \forall p \in P, \quad (9) \\
t^r_p & \geq 0, \quad \forall p, r \in P. \quad (10)
\end{align*}
\]

Notice that this formulation utilizes a special set of variables (keypath variables \( t^r_p \)), first proposed by Barnhart et al. (1995), to enhance model solution. Note also that the objective function minimizes spill plus carrying costs, or equivalently, assignment cost. Constraints (8) are the capacity constraints.

For leg \( i \), the term \( \sum_{p \in P} \sum_{r \in R} \delta^r_i t^r_p - \sum_{r \in R} \delta^r_i b^r_p t^r_p \) can be viewed as the number of passengers who are spilled from their desired itinerary \( p \). For leg \( i \), the term \( \sum_{p \in P} \sum_{r \in R} \delta^r_i b^r_p t^r_p - \sum_{r \in R} \delta^r_i b^r_p t^r_p \) is the number of passengers who are recaptured by the airline. (Note that we assume \( b^r_p = 1 \).) \( \text{CAP}_i \) is the capacity of the aircraft assigned to leg \( i \), and \( Q_i \) is the unconstrained demand on flight leg \( i \), which can be written mathematically as

\[ Q_i = \sum_{p \in P} \delta^r_i D_p. \quad (11) \]

Constraints (9) are the demand constraints that restrict the total number of passengers spilled from itinerary \( p \) to the unconstrained demand for itinerary \( p \). \( t^r_p \) must be greater than zero but need not be integer because we model the problem based on average demand data, which can be fractional.

**2.3.1. PMM Solution.** To solve PMM, the second set of constraints are relaxed initially. The rationale is that there can be many of these constraints, one for each itinerary, and the constraints are not likely to be binding in optimal solutions. To understand why, observe that the objective function coefficients are typically positive. Even though most of the time the fare of itinerary \( r \) is higher than that of itinerary \( p \), the actual fare collected is scaled down by the recapture rate, which can be a small number. Notice also that if \( r \) is the same as \( p \), i.e., the passengers are not redirected to any other itineraries, then the net effect on the objective function is zero. Since most of the objective function coefficients are positive, an optimal solution reduces \( t^r_p \) values as much as possible. As a result, constraints (9) will not be binding typically. Similarly, since most \( t^r_p \) values are zero or close to zero, we can further reduce the size of the problem by initially omitting all \( t^r_p \) except those for null itineraries, \( t^r_0 \). The rationale is that most of the passengers will be carried on their desired itineraries; only in cases where capacities are limited will spill and recapture occur.

**Column and row generation techniques** are then used to solve a restricted PMM, which initially ignores constraints (9) and all \( t^r_p \) variables except those for null itineraries, \( t^r_0 \). In column and row generation techniques, subsets of variables and constraints are neglected to create a restricted master problem. After
solving the restricted master problem, *pricing and separation subproblems* are solved to identify columns that could potentially improve the solution, and to identify violated constraints, respectively. These columns and constraints are added to the restricted master problem and the process is repeated until no columns or rows are generated. The solution approach for PMM is detailed in Kniker et al. (2001) and results are presented using data from a large United States airline.

### 3. Itinerary-Based Fleet Assignment Model

Enhancing the basic fleet assignment model has been of interest to the airline industry and researchers. Farkas (1995), Jacobs et al. (1999), and Rexing et al. (2000) develop enhanced fleet assignment models that are able to capture several interesting aspects of the problem not captured by FAM.

Farkas (1995) develops an itinerary-based fleet assignment model that combines the basic fleet assignment model with a passenger mix model ignoring recapture. He presents two different procedures for solving the model. The first procedure, a column generation approach, requires the repeated solution of the basic fleet assignment model because each decision variable in his model is a complete fleeting. To date, this approach is computationally impractical for problems of the size encountered in the industry. Farkas also suggests a second, heuristic approach that partitions the flight legs into subnetworks where network effects of multileg itineraries are present.

Erdmann et al. (1997) present a sequential approach in which a fleet assignment model is solved and then, given the resulting fleeting, a passenger mix problem is solved. They propose various solution approaches, including Lagrangian relaxation.

Jacobs et al. (1999) present a different approach to the origin and destination fleet assignment problem. In their implementation, a series of fleet assignment model relaxations and the combined fleet assignment and passenger flow problem are solved iteratively. Once prespecified criteria are met, the model is solved for the optimal integer solution.

Other improvements or extensions to basic FAM include, for example, Soumis et al. (1980), Ioachim et al. (1999), Jarrah and Strehler (2000), and Rexing et al. (2000).

#### 3.1. Formulation

Our Itinerary-Based Fleet Assignment Model (IFAM) integrates basic FAM and PMM. In IFAM, spill and recapture, and their associated costs, are decisions of the model. These decisions are constrained by other decisions, namely the capacity assigned to the network. In this way, we capture the networkwide interdependencies of capacity and revenue, thus improving on basic FAM.

By combining (FAM) and (PMM), we build upon the work of Farkas (1995), in which recapture is ignored. The objective of IFAM is to minimize assignment cost.

In IFAM, the variable definitions are the same as those in basic FAM and PMM except that $\text{CAP}_i$ is now replaced by $\text{SEATS}_k$, denoting the capacity of fleet type $k$. The first three sets of constraints (Equations (13) to (15)) are constraints of basic FAM and the next two sets of constraints (Equations (16) and (17)) are the constraints of PMM. Note however, that there is a change in one of the terms in the capacity constraints (16). Specifically, the first term on the left has been moved from the right-hand side because now, the capacity of the flight is also a variable:

$$\text{Min } \sum_{i \in I} \sum_{k \in K} \tilde{c}_{k,i} f_{k,i} + \sum_{p \in P} \sum_{r \in P} \left( \text{fare}_p - b_{p}^{t'} \tilde{c}_{p} \right) t'_p$$

subject to:

$$\sum_{k \in K} f_{k,i} = 1, \quad \forall i \in L,$$  

$$y_{k,o,t} - \sum_{i \in I(k,o,t)} f_{k,i} - y_{k,o,t} = 0, \quad \forall k, o, t,$$  

$$\sum_{o \in o} y_{o,t} + \sum_{k \in K} f_{k,i} \leq N_k, \quad \forall k \in K,$$  

$$\sum_{k \in K} \text{SEATS}_k f_{k,i} + \sum_{p \in P} \sum_{r \in P} \delta_{i} t'_p \geq Q_i, \quad \forall i \in L,$$  

$$\sum_{p \in P} t'_p \leq D_p, \quad \forall p \in P,$$
\[
\begin{align*}
 f_{k,i} & \in \{0,1\}, \quad \forall k \in K, \forall i \in L, \\
y_{k,o,t} & \geq 0, \quad \forall k, o, t, \\
t_p^r & \geq 0, \quad \forall p, r \in P.
\end{align*}
\]

(18)  (19)  (20)

3.2. Generalizing FAM

The IFAM formulation given in (12)–(20) can be seen as an “enhanced” or generalized basic fleet assignment model including carrying costs, recapture, and passenger flow conservation (network effects). To illustrate, consider the IFAM case where \( \sum_{i \in L} \text{fare}_p(i) \delta^r_p = \text{fare}_p \) for all \( p \in P \), carrying cost equals zero, and \( b_p^r = 0 \) for all \( p, r \in P \). Then, IFAM can be equivalently rewritten as:

\[
\text{Min} \sum_{i \in L} \sum_{k \in k} \tilde{c}_{k,i} f_{k,i} + \sum_{i \in L} \sum_{p \in P} \text{fare}_p(i) \delta^r_p t_p^r(i)
\]

subject to:

\[
\sum_{i \in L} f_{k,i} = 1, \quad \forall i \in L, \tag{22}
\]

\[
y_{k,o,t} + \sum_{i \in (k,o,t)} f_{k,i} - y_{k,o,t} - \sum_{i \in O(k,o,t)} f_{k,i} = 0, \quad \forall k, o, t, \tag{23}
\]

\[
\sum_{i \in A(k,o,t)} y_{k,o,t} + \sum_{\in C_l(k)} f_{k,i} \leq N_k, \quad \forall k \in K, \tag{24}
\]

\[
\sum_{\in k} \text{SEATS}_i f_{k,i} + \sum_{p \in P} \sum_{\in r} \delta^r_p t_p^r(i) \geq Q, \quad \forall i \in L, \tag{25}
\]

\[
\sum_{\in r} \delta^r_p t_p^r(i) \leq D_p, \quad \forall p \in P, \forall i \in L, \tag{26}
\]

\[
t_p^r(i) - t_p^r = 0, \quad \forall p, r \in P, \forall i \in L, \tag{27}
\]

\[
f_{k,i} \in \{0,1\}, \quad \forall k \in K, \forall i \in L, \tag{28}
\]

\[
y_{k,o,t} \geq 0, \quad \forall k, o, t, \tag{29}
\]

\[
t_p^r(i) \geq 0, \quad \forall p, r \in P, \forall i \in L. \tag{30}
\]

If constraints 27 are eliminated, the optimal spill decisions can be determined using the greedy spill estimation procedure described for basic FAM (§2.2). This implies that constraints (25), (26), and (30) can be eliminated and instead captured directly through spill cost coefficients in the objective function. Hence, the optimal solutions to basic FAM are the same as those for (21)–(30) with no carrying costs or recapture, and equations (27) eliminated, that is, conservation of passenger flows (network effects) ignored. IFAM, therefore, improves upon FAM by capturing network effects as well as recapture and carrying costs.

3.3. Recapture Rates

Determining the recapture rates for alternative itineraries is itself a difficult problem. The basis for calculating the recapture rates in these experiments is the Quantitative Share Index (QSI). This industry standard measures the “attractiveness” of an itinerary relative to the entire set of other itineraries (including competing airlines) in that market.

The sum of the QSI values corresponding to all itineraries (including competitors) in a market is equal to one. The sum of the QSI for one airline is an approximate measure of its market share for that specific market. Therefore, if the airline offers a passenger only one of their itineraries, it is effectively removing all of their other itineraries from this market. Let \( q_p \) denote the QSI value of itinerary \( p \). Let \( Q_m \) represent the sum of all QSI values in market \( m \) for the airline, i.e., \( Q_m = \sum_{\in p} q_p \). Then, the base recapture rate, \( b_p^r \) is:

\[
b_p^r = \begin{cases} 
1.0 & \text{if } p = r, \\
\frac{q_r}{1 - Q_m + q_r} & \text{otherwise.} 
\end{cases} \tag{31}
\]

The base recapture rate is a measure of the probability of accepting the alternative itinerary as long as fare and difference in departure time is not a factor. The attractiveness of the alternative is based on the time of day of departure, length of trip, and number of connections.

While the base recapture rate gives us a starting point, we modify it for similarities in departure (or arrival) times. If a passenger is offered an itinerary that has a similar departure and arrival time as the desired itinerary, he/she is more likely to accept the alternative. Therefore, the recapture rate should be higher if there are similar departure and arrival times, and lower if there are drastic differences in departure and arrival times.

The results of this process are rough estimates of the recapture rates. Accurately determining recapture rates is difficult, if not impossible. We show in §5 how sensitive the fleeting decisions from IFAM are to variations in recapture rates.
3.4. Overview of the Solution Approach
The overall IFAM solution approach is depicted in Figure 1. We construct a restricted master problem (RMP) excluding constraints (17) and spill variables that do not correspond to null itineraries. Next we apply a preprocessing step involving coefficient reduction, to tighten the IFAM LP relaxation. Details of this step will be covered later in this section. Then the LP relaxation of the restricted master problem is solved using column and row generation. Negative reduced cost columns corresponding to spill variables, violated constraints (17) and cuts (Step 6) are added to the RMP and the RMP is resolved until the IFAM LP relaxation is solved. Given the IFAM LP solution, branch-and-bound is invoked to find an integer solution. An optimal IFAM solution could be determined using a branch-and-price-and-cut algorithm in which columns and constraints are generated within the branch-and-bound tree. Because column generation at nodes within the branch-and-bound tree is nontrivial to implement using available optimization software, we instead employ a heuristic IP solution approach in which branch-and-bound allows column generation only at the root node.

3.5. Solving the LP Relaxation
Our models are tested and calibrated using data from a major United States airline. Table 9 describes the characteristics of our data sets.

In Table 10, we present the times to solve the LP relaxations of IFAM*, IFAM with added columns and rows, without cut generation (Steps 2 and 6) compared to those for solving the basic FAM LP relaxation. (All experiments are performed on an HP 9000 computer with one GB RAM, and solved using CPLEX version 6.0.2.) We also present the number of flights that have been partially assigned to more than one fleet type. The number of integer variables that are fractional in the optimal solution of the LP relaxation can be a good indicator of the amount of time that must be spent in the branch-and-bound tree. Indeed, as expected from the Table 10 results, our experiments show that the amount of time in the branch-and-bound tree for IFAM can be much greater than that for FAM. IP solution times for FAM range...
3.5.1. Strengthening the LP Relaxation Through Coefficient Reduction. One way to alleviate some of the fractionality in IFAM is to reduce all coefficients in Equations (16) for which \( \text{SEAT}_{X} > Q_i \). To illustrate the effect, consider four fleet types A, B, C, and D (Table 11), and flight leg \( i \) that contains passengers from two itineraries: (i) 100 connecting passengers paying $200 each, and (ii) 60 local passengers paying $125 each. Define the marginal cost of optimally assigning an additional seat to flight leg \( i \) as the least cost incurred when capacity is increased from \( X \) to \( X + 1 \) seats on a particular flight, ignoring aircraft integrality. From this definition, we can construct Table 12. Notice that fleet type C is not included in any combination because its operating cost is more expensive relative to other fleet types. In order to optimize revenue and fleeting decisions for flight leg \( i \), in isolation of other flight legs, we rank the passengers in order of decreasing fare and we assign passengers in order to flight leg \( i \) until the marginal cost of adding another seat for the \( j \)th passenger is either (a) greater than the revenue of that passenger; or (b) all available seats are assigned. In this example, all 160 passengers are assigned to the flight because the cost of offering the 160th seat is $113.97 and the 160th passenger pays $125.00. Hence, fleet combination 4 is optimal with 0.4118 plane of fleet type B and 0.5882 plane of fleet type D.

Applying our coefficient reduction process, we reduce the capacity of fleet type D to 160, the total number of passengers requesting seats on the flight. With this adjusted capacity for fleet type D, entries in Table 12 change to those shown in Table 13. Notice now that the optimal combination for this flight leg is at the upper boundary of combination 2, which offers 120 seats or a full plane of fleet type B. Offering the 121st seat incurs additional cost of $177.98 and the 121st passenger pays only $125.

As shown in Kniker (1998), our coefficient reduction scheme eliminates only fractional solutions and not optimal solutions. Notice, however, that coefficient reduction is much weaker if we consider recapture. With recapture, to ensure that no integer solutions are omitted, we compare the capacity to \( Q_i + \sum_{p \in P} \sum_{q \in P} \delta_p^q b_p^q f_q^i \) instead of just \( Q_i \). Typically the number of passengers that potentially can be recaptured

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Flights</th>
<th>Total # of Integral Variables</th>
<th>Solution Time (CPU sec)</th>
<th>Solution # of Integral Variables</th>
<th>Solution Time (CPU sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1N-3A</td>
<td>157</td>
<td>151</td>
<td>0.11</td>
<td>121</td>
<td>1.49</td>
</tr>
<tr>
<td>1N-4A</td>
<td>431</td>
<td>431</td>
<td>2.24</td>
<td>275</td>
<td>11.98</td>
</tr>
<tr>
<td>1N-6A</td>
<td>283</td>
<td>677</td>
<td>32.57</td>
<td>530</td>
<td>116.58</td>
</tr>
<tr>
<td>1N-9</td>
<td>2,044</td>
<td>1,634</td>
<td>722.24</td>
<td>1,385</td>
<td>2,989.14</td>
</tr>
<tr>
<td>2N-3A</td>
<td>173</td>
<td>160</td>
<td>0.15</td>
<td>83</td>
<td>1.66</td>
</tr>
<tr>
<td>2N-4A</td>
<td>485</td>
<td>437</td>
<td>1.79</td>
<td>293</td>
<td>15.78</td>
</tr>
<tr>
<td>2N-6A</td>
<td>877</td>
<td>857</td>
<td>29.85</td>
<td>450</td>
<td>171.33</td>
</tr>
<tr>
<td>2N-9</td>
<td>1,888</td>
<td>1,595</td>
<td>442.37</td>
<td>1,173</td>
<td>3,076.22</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Combination</th>
<th>Capacity</th>
<th>Fleeting Combination</th>
<th>Marginal Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1–108</td>
<td>A</td>
<td>$104.17</td>
</tr>
<tr>
<td>2</td>
<td>109–120</td>
<td>A and B</td>
<td>$62.50</td>
</tr>
<tr>
<td>3</td>
<td>121–144</td>
<td>B and D</td>
<td>$113.97</td>
</tr>
<tr>
<td>4</td>
<td>145–160</td>
<td>B and D</td>
<td>$113.97</td>
</tr>
</tbody>
</table>

Table 11 - Fleet Capacity and Operating Cost

<table>
<thead>
<tr>
<th>Fleet Type</th>
<th>Number of Seats</th>
<th>Operating Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>108</td>
<td>$11,250</td>
</tr>
<tr>
<td>B</td>
<td>120</td>
<td>$12,000</td>
</tr>
<tr>
<td>C</td>
<td>144</td>
<td>$16,250</td>
</tr>
<tr>
<td>D</td>
<td>188</td>
<td>$19,750</td>
</tr>
</tbody>
</table>

Table 12 - Marginal Cost of Additional Seat

<table>
<thead>
<tr>
<th>Combination</th>
<th>Capacity</th>
<th>Fleeting Combination</th>
<th>Marginal Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1–108</td>
<td>A</td>
<td>$104.17</td>
</tr>
<tr>
<td>2</td>
<td>109–120</td>
<td>A and B</td>
<td>$62.50</td>
</tr>
<tr>
<td>3</td>
<td>121–144</td>
<td>B and D</td>
<td>$113.97</td>
</tr>
<tr>
<td>4</td>
<td>145–160</td>
<td>B and D</td>
<td>$113.97</td>
</tr>
</tbody>
</table>

Table 13 - Marginal Cost of Additional Seat After Coefficient Reduction

<table>
<thead>
<tr>
<th>Combination</th>
<th>Capacity</th>
<th>Fleeting Combination</th>
<th>Marginal Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1–108</td>
<td>A</td>
<td>$104.17</td>
</tr>
<tr>
<td>2</td>
<td>109–120</td>
<td>A and B</td>
<td>$62.50</td>
</tr>
<tr>
<td>3</td>
<td>121–144</td>
<td>B and D</td>
<td>$177.08</td>
</tr>
<tr>
<td>4</td>
<td>145–160</td>
<td>C and D</td>
<td>$218.75</td>
</tr>
</tbody>
</table>
onto a flight leg is even greater than the unconstrained demand.

Table 14 compares the number of integral variables in the IFAM LP optimal solution with and without coefficient reduction, and no recapture. The coefficient reduction increases the number of integral variables in the LP solution by 6% to 36%.

3.5.2. Spill Partition Cut Generation. Let \( t_p = \sum_{r \in P} t_{p,r} \), then after solving an IFAM LP relaxation, we partition spill variables into two sets: those that are strictly positive (denoted \( T^+ \)) and those that are equal to zero (denoted \( T^0 \)). Without loss of generality, consider fleet type 1 with \( Q_i > SEATS_1 \). If \( \sum_{p \in T^+} \delta^p_i t_p \) is strictly positive and

\[
\sum_{p \in T^+} \delta^p_i t_p < Q_i - SEATS_1
\]

we want to ensure that we require more spill if the fleeting variable of fleet type 1 is strictly positive. The following spill partition cut can exclude a solution using fleet type 1 on flight leg \( i \) fractionally:

\[
(SEATS_1 - Q_i + \sum_{p \in T^+} \delta^p_i D_p) f_{1,i} + \sum_{p \in T^0} \delta^p_i t_p \geq 0.0. \tag{33}
\]

Inequality (33) ensures that for an assignment of fleet type 1 to flight \( i \), the total amount of potential spill from the set of spilled itineraries in the current LP solution on flight leg \( i \) plus spill on other itineraries containing \( i \) must exceed or equal the difference between unconstrained demand on leg \( i \) and the number of seats on aircraft of type 1. Because we could construct one such cut for each flight leg \( i \), for each fleet type \( k \in K \) assigned, and each potential partitioning of itineraries into sets with positive spill and sets with no spill, we do not initially include these cuts in (12)–(20). Instead we iteratively generate them by first solving the IFAM LP relaxation, and then partitioning the spill variables based on the LP solution, generating corresponding cuts for selected flight legs with fractional fleet assignments, and resolving the modified IFAM LP, and repeating the process.

In Table 15, assuming no recapture, we present the changes in the number of integral variables with and without inequalities (33). We also present the number of iterations that are performed before we stop adding these cuts. As the problem size gets larger, empirically we notice a diminishing effect with the addition of more cuts. There are a small number of flight legs with numerous itineraries, and many cuts are generated for these legs. These additional cuts, however, do little to improve performance. Hence, we establish a termination criterion that no additional cuts be generated when the improvement between two successive iterations is less than $1/day.

3.6. Solving the IP

Once the LP relaxation is solved, we use branch and bound to find an integer solution. Our approach is heuristic in nature as we do not generate columns within the branch-and-bound tree.
Our first phase of tree exploration is a depth first search to find a feasible solution. Through experimentation, we determined that the strong branching variable selection strategy in CPLEX achieves the best initial integer solution. A special branching strategy that branches on groupings of variables that form special ordered sets (SOS) is used to create the different branches. Specifically, the branching decisions are based on the cover constraints, which are of the form

\[ \sum_k f_{k,i} = 1, \]  

(referred to as Type 3 SOS constraints). Instead of branching by setting an individual variable to one and other variables to zero, a more effective branching strategy is to divide the set of variables into two sets, where the sum of the variables in the first set equals one or the sum of the variables in the second set equals one. This can lead to a near equal partitioning of the feasible solutions in the branching tree and, in practice, has resulted in some improvements in solution time (see Hane et al. 1995). Results of the branch and bound procedure are presented in Table 16.

Even though the IFAM solution determines both fleet assignments and passenger flows, the resulting passenger flows might not be optimal for the resulting fleet assignment because columns and constraints are not generated within the branch-and-bound tree. Hence, our approach to compute fleeting contribution (depicted in Figure 2) is to first solve IFAM (or FAM) and compute the total operating costs of the resulting fleet assignment. Then, given these fleeting decisions, we determine the minimum carrying and spill costs with PMM (allowing recapture). The difference in the unconstrained revenue and the sum of the operating, spill, and carrying costs is the fleeting contribution.

### 4. Analysis of IFAM Contribution

In this section, we measure the improvement of the IFAM solution over that of basic FAM. The analysis is prepared for results from runs on two full-size networks, i.e., 1N-9 and 2N-9. We attempt to explain the improvement in contribution attributable to: (a) network effects and (b) recapture.

To determine the independent impacts of network effects and recapture on fleeting contribution, we begin by solving basic FAM (Figure 3, Block i) using a mileage-based prorated fare allocation scheme (an industry standard approach, described in §2.2), and ignoring recapture. In order to quantify the impact of network effects, independent of recapture, we solve IFAM with no recapture (Figure 3, Block ii). To consider recapture we solve IFAM with recapture (Figure 3, Block iii). For each block, we apply the procedure in Figure 2 to obtain its associated fleeting contribution. In Figure 3, \( \alpha \) represents the improvement in fleeting contribution derived from capturing network effects and \( \beta \) represents the improvement in fleeting contribution derived from incorporating recapture into the fleeting process.
4.1. Results and Analysis

Tables 17 and 18 report the fleeting contributions associated with FAM (Block i), IFAM without recapture (Block ii), and IFAM (Block iii) with recapture.

The improvement ($\alpha$) in contribution from including network effects in the fleeting process ranges from $86,449 per day in problem 1N-9 to $104,864 per day in problem 2N-9, or, assuming schedule and demand are the same for the entire 365-day year, from $31.5 million to $38.3 million per year. The additional improvement ($\beta$) achieved by including recapture in the fleeting process ranges from $5,896 per day in problem 1N-9 to $314,901 per day in problem 2N-9, or $2.1 million to $115.0 million per year. The total improvement in IFAM’s fleeting contribution compared to FAM, thus ranges from $33.7 million for 1N-9 to $153.2 million per year for 2N-9.

In analyzing these differences, we observe that the operating cost is higher when the schedule is fleeted using FAM. This implies that in FAM, the demand for a given flight leg is overestimated and hence, too much capacity is assigned. This result is consistent with our observations that in FAM, network effects cannot be captured and passengers on multi-leg itineraries can be effectively spilled from one leg but not the others. The system load factor, that is, the ratio of the total number of passengers carried to the total number of seats supplied, is reported in Tables 17 and 18 for the FAM and IFAM solutions. By modeling network effects more accurately, IFAM clearly is better able to match passenger loads with capacity.

Next we observe in problem 2N-9 that IFAM with recapture compared to IFAM without recapture produces a fleet assignment generating more revenue and carrying fewer passengers (note that carrying cost per passenger is constant across all aircraft types). The average fare paid by passengers captured by the airline and the average fare of spilled passengers lost to the airline, reported in Table 19, is evidence that IFAM with recapture is better able to make fleeting decisions that allow high revenue passengers to be carried and low revenue passengers to be spilled.

### Table 17 The Improvement in the Fleeting Contribution for Data Set 1N-9 in Dollars per Day Compared to Basic FAM

<table>
<thead>
<tr>
<th>Changes in Fleeting Contribution</th>
<th>IFAM Without Recapture</th>
<th>IFAM with Recapture</th>
</tr>
</thead>
<tbody>
<tr>
<td>$+$86,449</td>
<td>$+$592,345</td>
<td></td>
</tr>
<tr>
<td>(Constrained) Revenue</td>
<td>$-0.55%$</td>
<td>$-0.70%$</td>
</tr>
<tr>
<td>Carrying Cost</td>
<td>$-1.00%$</td>
<td>$-1.34%$</td>
</tr>
<tr>
<td>Operating Cost</td>
<td>$-1.10%$</td>
<td>$-1.23%$</td>
</tr>
<tr>
<td>System Load Factor</td>
<td>$+32.44%$</td>
<td>$+32.44%$</td>
</tr>
</tbody>
</table>

### Table 18 The Improvement in the Fleeting Contribution for Data Set 2N-9 in Dollars per Day Compared to Basic FAM

<table>
<thead>
<tr>
<th>Changes in Fleeting Contribution</th>
<th>IFAM Without Recapture</th>
<th>IFAM with Recapture</th>
</tr>
</thead>
<tbody>
<tr>
<td>$+$104,864</td>
<td>$+$419,765</td>
<td></td>
</tr>
<tr>
<td>(Constrained) Revenue</td>
<td>$-0.69%$</td>
<td>$-0.07%$</td>
</tr>
<tr>
<td>Carrying Cost</td>
<td>$-1.28%$</td>
<td>$-1.67%$</td>
</tr>
<tr>
<td>Operating Cost</td>
<td>$-1.48%$</td>
<td>$-1.67%$</td>
</tr>
<tr>
<td>System Load Factor</td>
<td>$+24.52%$</td>
<td>$+23.83%$</td>
</tr>
</tbody>
</table>

### Table 19 The Changes in Average Fare Paid and Average Spill Fare Compared to Basic FAM

<table>
<thead>
<tr>
<th>Changes in Basic FAM</th>
<th>Average Fare Paid</th>
<th>Average Spill Fare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 1N-9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IFAM Without Recapture</td>
<td>$+0.08%$</td>
<td>$-3.34%$</td>
</tr>
<tr>
<td>IFAM with Recapture</td>
<td>$+0.12%$</td>
<td>$-4.49%$</td>
</tr>
<tr>
<td>Problem 2N-9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IFAM Without Recapture</td>
<td>$+0.33%$</td>
<td>$-5.94%$</td>
</tr>
<tr>
<td>IFAM with Recapture</td>
<td>$+1.28%$</td>
<td>$-14.62%$</td>
</tr>
</tbody>
</table>
5. **IFAM Validation**

In this section, we address several key assumptions built into IFAM, namely,

1. recapture rates,
2. deterministic demand, and
3. optimal passenger mix.

We show through empirical testing that IFAM needs only rough estimates of recapture rates. Fleeting decisions obtained using recapture rates that vary within a restricted range produce fleeting contributions that differ only slightly. Hence, IFAM consistently outperforms FAM for a range of recapture rates. As described in §2.2, the spill cost estimation in basic FAM recognizes certain stochastic elements in demand. In this section, we show through simulations that although IFAM does not consider demand uncertainty, its ability to model network effects and recapture allows it to outperform FAM. Next, we evaluate the sensitivity of our approach to the assumption that an optimal passenger mix is always achieved. We show through simulations that although IFAM does not consider demand uncertainty, its ability to model network effects and recapture allows it to perform well in a more realistic, less controlled environment.

### 5.1. Sensitivity of Fleeting Decisions to Recapture Rates

One of the most difficult parameters to estimate in IFAM is recapture rates because they are unobservable in reality. We do not need highly accurate estimates of recapture because, as our experiments show, the benefit of incorporating recapture into the fleeting decision process outweighs any errors that might result from inaccurate recapture rates.

In this exercise, we assume that the actual recapture rate from itineraries \( p \) to \( r \), is \( \hat{b}_p^r \), while \( b_p^r \) is the recapture rate from itineraries \( p \) to \( r \) used in IFAM. A fleeting decision is derived from IFAM using \( b_p^r \) as input, and this fleeting is then used to estimate contribution using \( \hat{b}_p^r \) in the PMM model. Figure 4 depicts this process. Sensitivity analysis is performed based on differing \( b_p^r \)'s, computed by multiplying \( \delta \) scaling factors and the actual recapture rate \( \hat{b}_p^r \), that is,

\[
b_p^r = \delta \hat{b}_p^r,
\]

(35)

To compare, we solve FAM and compute the estimated contribution using the PMM model with actual recapture rates, \( \hat{b}_p^r \). We perform this experiment on two small data sets, namely, 1N-3A and 2N-3A. The allowable integrality gaps for IFAM are set at $1,000 daily for both problems. The results are graphed and shown in Figures 5 and 6. We vary \( \delta \) from 0.5 to 1.5 at a 0.1 step. \( \delta \)'s are plotted on the horizontal axis. Note that \( \delta = 1.0 \) implies that the correct set of recapture rates are used in IFAM. The solid lines in Figures 5 and 6 show the improvement gained from considering network effects and recapture in IFAM for a given average daily demand. The dashed lines show the improvement gained from considering only network effects. Thus, the distance between the solid and dashed lines measures the benefit of recapture. Each diamond represents the improvement of IFAM over FAM when \( \hat{b}_p^r \) rates are used in IFAM. Thus, the distance from the solid line to each diamond measures the error from underestimating or overestimating the actual recapture rates by a factor of \( \delta \).

In Figure 5, the daily improvement of IFAM over FAM is approximately $7,500 with recapture and $5,600 without recapture. Thus, the daily benefit of considering recapture is $1,900. When very low recapture rates are used in IFAM, there are essentially no benefits to considering recapture in the fleeting process. When recapture rates are increased, the bene-
Figure 5  Recapture Rate Sensitivity for 1N-3A Data Set

The benefits of recapture remain relatively constant over a small range of values. As seen in Figure 5, the benefit increases. IFAM does not, however, appear to be particularly sensitive to the rates of recapture, over a small range of values. As seen in Figure 5, the benefits of recapture remain relatively constant over a range of \( \delta \), then jump to another level and, again, remain relatively constant. In Figure 6, the daily improvement of IFAM over FAM is approximately $47,500 with recapture and $30,000 without recapture. The daily benefit of considering recapture is $17,500. In this data set, we observe again that IFAM’s improvement relative to FAM increases as we increase recapture rates. The improvement reaches a certain level and remains relatively constant throughout. Note, however, that when \( \delta = 1.5 \) in Figure 6, a significant deterioration is observed. This is possible because the overestimated recapture rates are used when IFAM tries to optimize the fleeting, thus, IFAM would assign smaller aircraft to flight legs in an attempt to save operating costs anticipating that the spilled passengers are recaptured on other flight legs. When actual recapture is used to estimate contribution, however, the expected recaptured demand does not materialize, resulting in decreased profitability.

Although we perform these experiments on two relatively small problems, the problems are indicative of the results that would be achieved for larger problems. The conclusion we draw from this exper-
5.2. Demand Uncertainty

In this section, we measure the performance of IFAM compared to FAM in a simulated environment where realizations of forecasted demand vary by day of week. It is possible theoretically to solve IFAM for every day in a week and have different fleetings for the same flight on different days. This is, however, often impractical from an operational standpoint. Thus, airlines typically solve the fleet assignment model based on average demand data.

We compare how the fleeting decisions from FAM and IFAM compare using the methodology depicted in Figure 7. First, both FAM and IFAM are solved with average unconstrained demand. The resulting fleeting decisions are used to compute operating costs and are inputs to a simulator, in which we model the passenger allocation process. In this exercise, we use PMM with recapture as our allocation tool. Five hundred realizations of unconstrained itinerary demand are generated based on a Poisson distribution with the mean and standard deviation equal to the average unconstrained demand. Leg-level demand for FAM is computed by summing the appropriated itinerary-level demands. Leg-level demands are assumed again to normally distributed (as described in §2.2).

The simulations are performed on two data sets, namely, 1N-3A and 2N-3A. Table 20 summarizes the results. From Table 20, IFAM outperforms FAM on both data sets even though IFAM does not take into account demand uncertainty in its decision process. This suggests that the benefit of considering network effects and recapture might outweigh FAM’s ability to incorporate demand uncertainty at a flight leg level. Notice that in Problem 1N-3A, IFAM achieves the improvement almost solely from the increased revenue resulting from a better fleet assignment, while in Problem 2N-3A, IFAM achieves the improvement primarily through savings in the operating costs. In fact, in Problem 2N-3A, IFAM’s revenue decreases. In both situations, IFAM clearly produces fleet assignments that are superior to those of FAM.

This simulation uses PMM, assuming full control of passengers in allocating passengers to flight legs. In order to mimic more closely the actual environment where airlines do not have full control of passengers, we perform another simulation in which there is imperfect control of passengers in allocating them to itineraries.

5.3. Imperfect Control of Passenger Choices

In this section, we repeat the algorithm depicted in Figure 7; however, we modify the passenger allocation simulation to account for the airline’s inability to optimize passenger flows. Specif-

<table>
<thead>
<tr>
<th>Table 20</th>
<th>Comparison of the Performance of FAM and IFAM in Simulation 1 ($/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FAM</td>
</tr>
<tr>
<td>Problem 1N-3A</td>
<td></td>
</tr>
<tr>
<td>Revenue</td>
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<tr>
<td>Operating Cost</td>
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<tr>
<td>Contribution</td>
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<tr>
<td>Problem 2N-3A</td>
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</tr>
<tr>
<td>Revenue</td>
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</tr>
<tr>
<td>Operating Cost</td>
<td>$2,255,254</td>
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<tr>
<td>Contribution</td>
<td>$1,271,368</td>
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</table>
ically, we impose two restrictions on the passenger mix model to capture more closely the actual environment:

1. a 95% maximum load factor is allowed on all flights, and
2. a 70% maximum spill ratio is allowed on all itineraries.

With the first restriction, we attempt to account for the no-show phenomenon, in which passengers book tickets but are not present when the flight takes off, resulting in empty seats and potentially lost revenue to the airlines. The maximum allowable spill restriction is imposed in order to allow for imperfections in the revenue management process. By limiting the maximum itinerary spill to 70% of the unconstrained average demand, the resulting mix of passengers might more closely match the mix of passengers actually realized. The results from this simulation are presented in Table 21.

While contributions of FAM and IFAM in Table 21 are less than the corresponding contributions in Table 20 (as expected), IFAM still outperforms FAM by a significant margin in this simulated environment. It is unclear, however, whether the optimal passenger mix assumption has positive or negative impact on IFAM because the improvement in Problem 1N-3A deteriorates in Table 21, compared to Table 20, but in Problem 2N-3A, the improvement improves significantly.

Acknowledgments

<p>| Table 21 Comparison of the Performance of FAM and IFAM in Simulation 2 ($/day) |
|-----------------|-----------------|-----------------|</p>
<table>
<thead>
<tr>
<th>FAM</th>
<th>IFAM</th>
<th>Difference (IFAM – FAM)</th>
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</thead>
<tbody>
<tr>
<td><strong>Problem 1N-3A</strong></td>
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<td>Revenue</td>
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<td>Contribution</td>
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<td>$2,852,998</td>
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<tr>
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<tr>
<td>Revenue</td>
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<td>Operating Cost</td>
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<tr>
<td>Contribution</td>
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<td>$1,268,977</td>
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</table>

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References


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