A DECISION SUPPORT FRAMEWORK FOR MULTI-FLEET ROUTING AND MULTI-STOP FLIGHT SCHEDULING

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Abstract—Fleet routing and flight scheduling are important in airline carrier operations. Ineffective and inefficient fleet routing and flight scheduling will result in a substantial loss of profits. This research aims at developing a framework to help carriers adjust their draft timetables and fleet routes, when market demand conditions are expected to change in the near future. The framework is based on a basic model, constructed as a multi-fleet time-space network from which several strategic models are developed, to help carriers in fleet routing and flight scheduling. These models are formulated as multiple commodity network flow problems. The Lagrangian relaxation accompanied by the network simplex method, a Lagrangian heuristic and a modified sub-gradient method are suggested to solve the problems. A flow decomposition algorithm is also suggested in order to trace every aircraft route. To show how to apply this framework in the real world, a case study regarding the international operations of a major Taiwan airline was performed. The results show that the framework would be useful for actual operations. Copyright © 1996 Elsevier Science Ltd

1. INTRODUCTION AND LITERATURE REVIEW

Fleet routing and flight scheduling are important in airline carrier operations. In short-term operations, the market demand conditions may change due to such factors as the seasonal variation of traffic demands, new airlines entering the market, or a global change of market conditions. Airline carriers have to modify their flight timetables and fleet routes according to a projected market demand in the near future, in order to dynamically maximize their profits. Typically, the flight scheduling process consists of two phases: a schedule construction phase and a schedule evaluation phase. The construction phase is completed by drafting a timetable. In the schedule evaluation phase, the timetable draft is then examined by operating personnel for feasibility and other cost and performance considerations. Any desired improvements are then fed back into the construction phase, and a revised timetable draft is determined. The flight scheduling process iterates between these two phases until a satisfactory final timetable is obtained (Etschmaier & Mathaisel, 1984). According to a major airline carrier in Taiwan, as shown in Fig. 1, the current practice for fleet routing and flight scheduling is a trial-and-error process. First, planners propose a draft timetable based on factors such as the predicted market demand and its market share, the available airports, the slot times in each airport, the available airplanes (which are typically fixed in short-term operations), aircraft routes, etc. They then design fleet routes by considering the operating constraints, such as available airplanes, maintenance rules and crew availability. Through local aircraft routing, the adjusting of flight departure times within the available slots, the deletion of some ineffective flights or the renting of aircraft, the process is repeated, by hand, until a satisfactory solution is found. This approach, however, is inefficient for arranging a large flight network, and could possibly result in an inferior feasible solution.

This research aims at developing a decision support framework based on several strategic models in order to assist carriers in multi-fleet routing and multi-stop flight scheduling. This framework, as shown in Fig. 2, differs from the traditional process by its use of several strategic models. These strategic models are obtained from a basic model (referred to as a “flight deletion model”), which is used to evaluate the deletion of uneconomic flights. All the models consider a given draft timetable, a set of available airplanes, airport slots, airplane rental costs and other cost data as inputs, to be efficiently solved for maximum
profit. Carriers can choose a strategic model with inputs to systematically optimize their fleet routes and flight schedules. After comparing results from different strategic models, they can decide on the best fleet routes and timetables. As a result, the heart of the framework is the question of how to create the basic model and the strategic models for fleet routing and flight scheduling.

Fleet routing and flight scheduling models have been a subject of research for decades and were usually formulated as two types of integer network flow problems, specifically pure network flow problems and network flow problems with side constraints (NFPWS), with numerous types of objective functions and constraints. Since most of the models were developed for carrier use, the objective was usually taken as either the minimization of the fleet size (or system cost), or the maximization of system profits (Levin, 1969; Simpson 1969; Hyman & Gordon, 1978; Levin, 1971; Teodorovic, 1988). As to solution methods, pure network flow problems have been solved by minimum cost flow algorithms, for example, the Out-of-Kilter algorithm (see Simpson, 1969). Since integer network flow problems with side constraints are characterized as NP-complete problems (Garey & Johnson, 1979), typical exact solution techniques have been applied, such as the branch-and-bound method, the cutting plane method, etc. (Levin, 1969; Levin, 1971; Teodorovic & Guberinic, 1984; Lec, 1986; Teodorovic, 1988). To prevent the long computation time in applying exact IP solution techniques, Lee (1986) tried a Lagrangian technique to solve a single fleet routing problem. Unfortunately, the convergence results were far from satisfactory. The results showed that, with the subgradient method (Fisher, 1981), it was difficult to find good Lagrangian multipliers that improved the lower bound. As a result, Lee applied a branch-and-bound algorithm instead.

In the literature, Simpson’s result (1969) is more related to our work than the others. Simpson (1969) applied the time–space network technique to the development of various models that consider different routing factors, such as multiple flight departure times, various objectives, etc. Although Simpson proposed a very useful modelling technique for the fleet routing problem, there are a number of limitations in its use for airline carriers in Taiwan or other areas. Most importantly is that the problems solved by Simpson (1969) were single fleet routing problems. For multi-fleet routing problems, Simpson only stated a draft multi-commodity network model dealing with non-stop flights. There was no discussion of model modifications for various routing strategies, such as changing flight departure times and evaluating multi-stop flights. Besides, there was neither a solution algorithm developed

![Fig. 1. Traditional fleet routing and flight scheduling process.](image-url)
nor any tests of real data for multi-fleet routing problems. Consequently, the multiple fleet routing problem has remained unsolved. As to solution techniques for single fleet routing problems, Simpson (1969) applied a Lagrangian multiplier technique, and Dantzig–Wolfe decomposition to solve a single fleet network with few side constraints. Although the convergence by the Lagrangian multiplier technique was good, the solution obtained was not necessarily a feasible one for the NFPWS (our case study in Section 4 also shows this), but theoretically, a lower bound. Similarly, because Dantzig–Wolfe decomposition is a linear programming solution technique, it will, in general, give an optimal convex combination of the integer sub-problem solutions which will not be an integer itself (Simpson, 1969; Yan, 1991). Thus, the approximation approaches proposed by Simpson (1969) are not used in our research.

Besides these limitations, we note that Simpson's models can be modified or extended to apply to other practical strategies which are suitable for Taiwan carriers; for example: (1) because many international flights for Taiwan carriers are long-haul flights, there are always cross-cycle flights when a circulated time-space network is applied; (2) the changing of flight departure times may involve the use of a carriers' own slots or other carriers' slots subject to negotiation; and (3) a one-stop flight, typically serving three OD demands, may be evaluated for three, instead of two, legs; all of these were not modelled by Simpson (1969). Furthermore, aircraft renting is a practical strategy for short-term operations. It would be useful for a carrier to understand whether it would be profitable to rent airplanes, and even, to decide how many airplanes should be rented when market demand conditions are expected to change in the near future.

It should be mentioned that some other types of airline scheduling models have been recently developed by, for example, Abara (1989), Teodorovic and Krcmar-Nozic (1989),

![Diagram](image-url)  
Fig. 2. A new framework for fleet routing and flight scheduling.
Balakrishnan et al. (1990) and Jarrah et al. (1993). In particular, Abara (1989) developed an integer linear programming model to solve the fleet assignment problem. Teodorovic and Krcmar-Nozic (1989) developed a multicriteria model to determine flight frequencies on an airline network under competitive conditions. Balakrishnan et al. (1990) introduced a mixed integer program model to select aircraft routes for long-haul operations, which is close to the schedule construction phase. Jarrah et al. (1993) proposed two minimum cost network flow models to systematically adjust aircraft routing and flight scheduling in real time in order to minimize the total cost incurred under the shortage of aircraft. However, none of the problems solved in these models is the same as ours. In summary, multiple fleet routing and flight scheduling problems were not modelled and solved in practice. It is believed that the modelling of multi-fleet routing problems especially incorporating various strategies, is much more complicated than that of the existing single-fleet routing models. Since there are multi-type aircraft in most airline operations, it is useful for such airline carriers (for example, most airline carriers in Taiwan) to develop “strategic” multi-fleet routing models as well as an efficient algorithm to solve them. This is, indeed, the focus of this research.

From the review, the time-space network technique, which has recently become popular in the field of logistics, is both a natural and flexible method for modeling fleet routing problems. Only a few nodes, links, or additional side constraints need to be modified, without changing the original network structure (Simpson, 1969; Yan et al., 1995). If the problem is formulated as an NFPWS, then after relaxing the side constraints, the model becomes a minimum cost flow problem where there are efficient minimum cost flow algorithms applicable for finding solution bounds. Therefore, if the Lagrangian sub-problem can provide a good lower bound (for minimization problems), then, accompanied by a good heuristic for finding a feasible solution as well as a good subgradient method to modify the Lagrangian multipliers, the LRS could be useful, due to its fast convergence in actual practice (Fisher, 1981; Fisher, 1985; Balakrishnan et al., 1990; Powell & Ioannis, 1992). In addition, being able to divide a major problem into several sub-problems, LRS can save memory space during computation, which is useful for arranging large-size networks.

This research will apply the time-space network technique to formulate multiple fleet routing and flight scheduling problems. We develop a Lagrangian-based algorithm to solve these models, where a good Lagrangian heuristic and a good sub-gradient method are essential to the algorithm’s performance. This research will focus on one-stop and non-stop flight operations. Although there is a significant interdependence between the airline schedule design process and the aircraft maintenance, as well as the crew scheduling processes, these processes are usually separated in order to facilitate problem solving (Teodorovic, 1988). This research thus excludes the constraints of aircraft maintenance and crew scheduling in the modelling. In practice, the fleet routes and flight timetables obtained from this framework can serve as a good initial solution for minor modifications of these soft constraints. However, actually incorporating these constraints into fleet/flight scheduling would be a topic for future research.

The rest of this paper is organized as follows: we first introduce the modelling approach, including a basic model and several strategic models. After formulating these models as integer multiple commodity network flow problems, we then develop a solution algorithm to solve them. Finally, we perform a case study to test the framework.

2. MODELLING APPROACH

The framework proposed in this research is based on a basic multi-fleet model from which several strategic models are developed. The basic multi-fleet model contains several single-fleet time-space networks, each formulating a single fleet routing in a given period. In this section, we first introduce the single-fleet time-space network, and then extend it to the basic multi-fleet model. We also develop a number of other strategic models.

The single-fleet time-space network

This research suggests using a time-space network to formulate a single-fleet routing problem as shown in Fig. 3. The horizontal axis represents airport locations; the vertical
axis represents the time duration. In general, fleet routing is rotated periodically, in particular after 1 week for most carrier operations. To ensure the circulation of fleet routing across consecutive cycles, the time line should be continued from the end, for each station, back to the beginning. The network flows represent the flow of airplanes in the dimensions of time and space. Two major components, nodes and arcs, are contained in the network. A node represents an airport at a specific time, while an arc represents an activity, for example, a flight, a ground-holding, an overnight stay, etc. Four types of arcs are defined below.

(i) Flight arc. A flight arc represents a non-stop flight or a one-stop flight. Flight arcs are designed based on a draft timetable which is composed of all drafted flights. Each flight arc contains information about the departure time, the departure airport, the arrival time, the arrival airport, the operating cost and the revenue. The block time for a non-stop flight is calculated from the time when the airplane is prepared for this flight to the time when this flight is finished. If the flight is a multiple-stop flight, then the block time is from the beginning of the first leg to the end of the last leg. The block time thus includes the time for investigation before departure, fuelling, passenger/baggage boarding and deplaning, and air flight time. It should be noted that, if there are flights across 2 consecutive weeks, then the flight arcs should be formulated as is (six) in Fig. 3. The arc cost is equal to the flight cost minus passenger revenues. The arc flow upper bound is one, meaning that the flight can be served at most once. The arc flow lower bound is zero, indicating that this flight can be deleted.

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Fig. 3. The single-fleet time-space network.
(ii) Ground arc. A ground arc represents the holding of airplanes at an airport in a time window. The arc cost, including airport tax, airport holding charge, gate use charge, etc., represents the expenses incurred for holding an airplane at an airport in the corresponding time window. The arc flow upper bound is the apron capacity (or infinity, if the capacity is large), representing the maximum number of airplanes that can be held at this airport during a specific time window. The arc flow lower bound is zero, meaning that no airplanes are necessarily held at this airport in this time window.

(iii) Overnight arc. An overnight arc represents the holding of airplanes overnight. The time window is set for the overnight duration between two consecutive days. The arc cost is the cost for an airplane held overnight at the airport. The arc flow upper bound and lower bound are the same as those of the ground arcs.

(iv) Cycle arc. A cycle arc represents the continuity between two consecutive planning periods. It connects the end of one period to the beginning of the next period for each airport. The arc's parameters are the same as those of the overnight arcs.

The basic multi-fleet model

The basic multi-fleet model (BMFM) is represented as multiple single-fleet time-space networks; each is built as mentioned above. An example of the BMFM is shown in Fig. 4, assuming that there are three types of fleet in operation, where the capacity of type A is the...
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The smallest and that of type C is the largest. Since the larger aircraft can serve smaller-type flights, the smaller-type drafted flight arcs can be added into the larger-type fleet networks. For example, as shown in Fig. 4, the type A flight arc drafts can be added into the type B and the type C networks; while the type B flight arc drafts can only be added into the type C network. Thus the type C fleet can serve not only its draft flights but also the type B and the type A draft flights. Similarly, the type B fleet can serve the type A draft flights as well as its own. As a result, the density of the type C network is higher than that of the B-type network which is higher than that of the type A network.

Note that there is no node supply/demand in each fleet network. To ensure that the number of required airplanes in a routing does not exceed the number of available airplanes for each fleet, and that each flight can be served at most once, two types of side constraints are added in the problem formulation: (1) the sum of all cycle arc flows and flight arcs across consecutive weeks in each single-fleet network is not greater than the number of available airplanes; (2) the sum of all arc flows corresponding to the same flight should be less than or equal to one. The objective of this model is to "flow" all the airplanes in all single-fleet networks at a minimum cost. Although the revenue for serving flights is put in the form of a negative cost, the objective is equivalent to obtaining a maximum profit. The BMFM can be formulated as an integer program as follows:

\[
\text{min } \sum_{n \in M} \sum_{i \in A^n} C^n_{ij} X^n_{ij}
\]

\[
\text{s.t. } \sum_{j \in N^n} X^n_{ij} - \sum_{k \in N^n} X^n_{ki} = 0, \quad \forall i \in N^n, \quad \forall n \in M
\]  

\[
\sum_{(i,j) \in P^n} X^n_{ij} + \sum_{(i,j) \in U^n} X^n_{ij} \leq P^n, \quad \forall n \in M
\]  

\[
\sum_{n \in M} X^n_{ij} \leq 1, \quad \forall (i,j) \in A^n
\]  

\[
0 \leq X^n_{ij} \leq U^n_{ij}, \quad \forall (i,j) \in A^n, \quad \forall n \in M
\]  

\[
X^n_{ij} \in \text{integer}, \quad \forall (i,j) \in A^n, \quad \forall n \in M
\]  

where \( n \) = the \( n \)th fleet; \( M \) = the set of all fleets; \( N^n \) = the set of all nodes in the \( n \)th fleet network; \( A^n \) = the set of all arcs in the \( n \)th fleet network; \( A^n_i \) = the set of all flight arcs in the \( n \)th fleet network; \( P^n \) = the set of all cycle arcs in the \( n \)th fleet network; \( U^n \) = the set of all arc flows across weeks in the \( n \)th fleet network; \( C^n \) = cost of arc \((i,j)\) in the \( n \)th fleet network; \( P^n \) = number of available airplanes in the \( n \)th fleet network.

Model (2.1) is an integer multiple commodity network flow problem; in which the objective is to minimize the system cost, constraint (1) ensures flow conservation at every node in each fleet network, eqn (2) denotes that the number of airplanes used in each fleet routing is not greater than that of the available airplanes, eqn (3) indicates that each flight can be served at most once, eqn (4) ensures that all arc flows are within their bounds, and eqn (5) ensures the integrality of airplane flows.

The strategic models

To develop strategic models based on the BMFM, as shown in Fig. 5, we suggest using three basic strategies, in particular, (a) deletion of the drafted multi-stop flight legs, (b) adjustment of flight departure times, and (c) aircraft rentals. For example, in Fig. 5, strategy "d" combines strategy (a), (b) and (c), and a systematic fleet routing. Users can evaluate results from various strategic models to choose the best strategy. Modifications of the BMFM for each basic strategy are described below.

(a) Deletion of multi-stop flight segments. To improve a system flight's continuity and fleet routing, some uneconomic segments in some multi-stop flights can be suitably deleted. To evaluate the deletion of one-stop flight segments, we suggest an extension of the modelling...
technique given by Simpson (1969). As shown in Fig. 6, $X_{ij}$, $X_{ik}$ and $X_{jk}$ stand for a non-stop flight from $i$ to $j$; from $j$ to $k$; and from $i$ to $k$, respectively. $X_{ijk}$ stands for a one-stop flight from $i$ to $j$ to $k$. Note that $X_{ik}$ is an addition in the technique proposed by Simpson (1969). As introduced by Simpson (1969), two non-stop segments ($X_{ij}$ and $X_{ik}$) are bridged by a one-stop arc ($X_{jk}$). A dummy node ($a$ or $b$) is placed on flight $ij$ and flight $jk$. This construction allows either $ij$ or $jk$ segments to be operated independently as non-stop flights. Any previous arrivals at $j$ cannot use the attractive “one-stop” arc joining the two flights. There are two cost arcs created for use if the flights operate independently. If the flights are connected, the $X_{ij}$ flow directly transfers to the $X_{jk}$ flow and receives an additional benefit of $R_{ik}$ for doing so. This modelling, however, does not consider the passenger demand from $i$ to $k$.

To evaluate the decomposition of a one-stop flight into complete non-stop flights, this research introduces another non-stop segment, $X_{ik}$, and an additional side constraint to the aforementioned modelling. As a result, there are three side constraints added to model (2.1). Firstly, since the flight segment from $i$ to $k$ can be chosen at most once, that is, the sum of $X_{ik}$ and $X_{jk}$ can be, at most, 1, an additional constraint, $\sum_{n \in M} (X_{ik} + X_{jk}) \leq 1$, should be added to eqn (3) of model (2.1). If $X_{ik}$ has flows (should be 1), then $X_{jk}$ should be 0, meaning that the one-stop flight from $i$ to $j$ to $k$ is not served. Similarly, if $X_{jk}$ has flows (should be 1), then $X_{ik}$ should be 0, meaning that the passenger demand from $i$ to $k$ has been served by the one-stop flight $i$ to $j$ to $k$, and there is no need to provide the non-stop flight from $i$ to $k$. Secondly, since a flight from $i$ to $a$ (i.e. a non-stop flight from $i$ to $j$) and from $b$ to $k$
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(i.e. a non-stop flight from \textit{j} to \textit{k}) can be served, at most once, two side constraints, \(\sum_{n \in M} X_{ik}^n \leq 1\) and \(\sum_{n \in M} X_{ik}^n \leq 1\), should also be added to eqn (3). Here, if there are one-stop flights across 2 weeks, then constraint (2) should be modified as \(\sum_{(i,j) \in E} X_{ij}^n + \sum_{(j,b) \in U} X_{jb}^n + \sum_{(i,k) \in U} X_{ik}^n \leq F^n\).

Note that the arc cost of \(X_{ik}\) is the flight cost minus the passenger revenue obtained on the non-stop flight from \textit{i} to \textit{k}; all other arc costs are set to be the same as those in Simpson (1969). Notice that, besides \(X_{ik}\), these three side constraints were not modelled in Simpson (1969). Modifications for two-stop or more-stop flights can be researched in the future.

(b) Adjustment of flight departure times.

To evaluate the best departure time for a flight, a carrier can add alternate flight arcs (as shown in Fig. 7), representing different departure times. For example, \(F_1\) denotes a flight, while \(F_2\) and \(F_3\) denote alternate flights. If \(F_2\) or \(F_3\) contains a flow in the optimal solution, it means that the departure time of \(F_1\) should be moved to a later time. The parameters for these alternate arcs are those of the flight arcs.

Because at most one departure time is assigned to a flight, a side constraint should be introduced for a flight arc and its alternate flight arcs. Assuming that \(X_{ij}^n\) is the flow of the flight arc \((i,j)\) in the \(n\)th network and \(b_1\) is the set of the drafted flight arc and their associated alternate flight arcs, then a side constraint, \(\sum_n \sum_{(i,j) \in b_1} X_{ij}^n \leq 1\), should be added to eqn (3) in model (2.1).

Note that carriers can set the number of alternate flight arcs for each flight based on their operations or other concerns, for example, slot time constraints. The extreme case has full sliding arcs within the slot time for every flight. Certainly, when more flights are evaluated for departure times or more alternate flights are added, better results are expected, while the problem size grows. Carriers may trade off the problem size and the system's objective.

It should be mentioned that, though the aforementioned candidate flight departure times are located in the carrier's own slots, the strategy can be extended to the evaluation of departure times within another carrier's slot, if its own slots are not enough, and if it is feasible to negotiate with other carriers for renting or exchanging slots. Carriers can evaluate whether it is profitable to rent slots from others or to exchange slots with others. The modifications for this extension, as shown in Fig. 8, are similar to those in Fig. 7, but with additional alternate arcs within another carrier's slots. Rental costs should be added to these additional alternate arcs if one is to evaluate slot rentals. If the carrier must evaluate whether to exchange its slots with other carriers, then no rental cost is needed. The optimal solution will determine how long and when the carrier should rent slots or exchange slots with others to obtain a maximum profit. Note that the rental cost increases more when the alternate arcs are far away from the carrier's own slots, meaning that larger slots are considered for rental. With this modification, the objective function should include this rental cost. The side constraints mentioned on these alternate flight arcs are modified as \(\sum_n \left( \sum_{(i,j) \in b_1} X_{ij}^n + \sum_{(i,j) \in b_2} X_{ij}^n \right) \leq 1\); where, similar to \(b_1\) mentioned above, \(b_2\) is the set of the associated alternate flight arcs in the negotiable slots.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig7.png}
\caption{Network modifications for adjustment of a flight’s departure time.}
\end{figure}
(c) Aircraft rentals. If demand increases and the current available aircraft cannot handle most draft flights through efficient fleet routing and flight scheduling, carriers may consider aircraft rentals to improve their services and system profits. We suggest a strategy to evaluate what type and how many airplanes are suitable for rental. As shown in Fig. 9, the modification adds a rental arc for each station, for each fleet network. The arc flow indicates the number of aircraft rentals for a fleet. It is formulated similar to a cycle arc, except that its arc cost is equal to the average charge for renting an airplane for a week. If the aircraft rental contract period is longer than the planning span, then carriers may set a different weekly rental cost from the average to reflect the seasonal fluctuations of market demand conditions. Carriers can determine whether to rent aircraft based on results, their long term policies, etc. Note that, for this modification, no side constraints are added to the BMFM, and only some flow conservation constraints related to additional rental arcs are modified. However, if the number of total rental airplanes (for a fleet or for all fleets) is constrained by the carrier’s policy, government regulation, or other reasons, a side constraint ensures that the total rental arc flows (in a fleet network or in all fleet networks) should be added to the BMFM to prevent the rental of too many airplanes.

3. SOLUTION METHOD

The basic model and the strategic models are formulated as integer multiple commodity network flow problems which are characterized as NP-complete problems (Garey & Johnson, 1979). From the literature review in Section 1, this research suggests using the LRS for the approximation of near-optimal solutions. For the convenience of later explaining the LRS, let us use a general eqn (6), to stand for constraints (2) and (3) in formula (2.1), and rewrite (2.1) as follows:

\[
\begin{align*}
\min \quad & \sum_{n \in M} \sum_{i,j \in A^n} C_{ij}^n X_{ij}^n \\
\text{s.t.} \quad & \sum_{j \in N^n} X_{ij}^n \sum_{k \in k_i N^n} X_{ki}^n = 0, \quad \forall i \in N^n, \quad \forall n \in M \\
& \sum_{(i,j,n) \in H_s} X_{ij}^n \leq \delta_s, \quad \forall S \\
& 0 \leq X_{ij}^n \leq U_{ij}^n, \quad \forall (i,j) \in A^n, \quad \forall n \in M \\
& X_{ij}^n \in \text{Integer}, \quad \forall (i,j) \in A^n, \quad n \in M,
\end{align*}
\]

where \( H_s \) is the \( s \)th side constraint set of arcs in all networks. \( \delta_s \) is the right-hand-side associated with the \( s \)th set. The LRS applied in this research is summarized in three parts.

![Fig. 8. Network modifications for adjustment of flight departure times with negotiable slots.](image-url)
A lower bound

The side constraints (6) are relaxed with a non-negative Lagrangian multiplier \( u = (u_1, u_2, \ldots, u_n) \) and are added to the objective function of formula (3.1), resulting in the following program:

\[
\begin{align*}
\min & \quad Z = \sum_{neM} \sum_{i \in A^n} C_{ij} X_{ij} + \sum_i u_i \left( \sum_{i,j \in H_i} X_{ij} - \delta_i \right) \\
\text{st} & \quad \sum_{j \in N^n} X_{ij} - \sum_{k \in N^n} X_{ki} = 0, \quad \forall i \in N^n, \forall n \in M \tag{1} \\
0 & \leq X_{ij} \leq \ell_{ij}, \quad \forall (i, j) \in A^n, \forall n \in M \tag{4} \\
X_{ij} & \in \text{Integer}, \quad \forall (i, j) \in A^n, \forall n \in M \tag{5}
\end{align*}
\]

\[ u_i \leq 0, \quad \forall s. \]

Obviously, formula (3.2) can be decomposed into several independent, pure network flow problems. We suggest using the network simplex method to solve these pure network flow problems due to its efficiency (Kennington & Helgason, 1980; Ahuja et al., 1993). The optimal objective value of formula (3.2) is a lower bound for formula (3.1) (Fisher, 1981).

Fig. 9. Modifications of the aircraft rental network.
An upper bound

A Lagrangian heuristic is developed to find a feasible solution and an upper bound, from the lower bound solution (usually an infeasible flow) found in each iteration. Since the optimal solution of the Lagrangian problem generally satisfies most constraints, it is not “far” from the optimal solution (Fisher, 1981). We optimally solve the Lagrangian problem and then apply a systematic approach to “perturb” the infeasible solution to be feasible. The steps of the heuristic are as follows: if a side (so called bundle) constraint is violated, then there should be more than one arc in this set of bundle arcs with positive flows. We choose the arc from this bundle with the largest arc cost (after being modified by LR), then reduce the arc flow from one to zero.

In order to maintain flow conservation, we find a least cost flow augmenting path from this arc’s tail to the arc’s head by a modified label correcting algorithm (Powell, 1989), and augment a unit of flow through the path; thus we reduce an extra unit of flow on the side constraint, surely increasing the objective value. It should be noted that, since any arc flow involved with side constraints cannot be increased during flow augmentation to prevent the violation of other side constraints, we set all flight arc flows as their flow upper bounds in the flow augmentation. If the side constraint is not yet satisfied, we choose another arc, with the second largest arc cost in this bundle, to reduce another unit of extra flow. The process is repeated until this side constraint is satisfied. We then examine another side constraint and make it feasible. When all the side constraints are scanned and modified to become feasible, we find a feasible flow.

Interestingly, this heuristic always finds a feasible solution (an upper bound). First, for any flight arc with a positive flow (equal to 1), by the flow decomposition theory (Ahuja et al., 1993), there exists a circuit, say CC+, with a positive flow (equal to 1) passing this arc. Then, in the flow augmentation for this flight arc, we can find at least a circuit CC, which is the reverse of CC+. When we augment a unit of flow in CC, we reduce all arc flows in CC by one unit. All flight arc flows in CC then become zero, and no other side constraints are violated in this flow augmentation. Thus, we can always reduce an extra unit of flow when we examine a flight arc on a specified side constraint. Finally, we can augment extra units of flows for all side constraints and obtain a feasible solution. Notice that the extreme case for the heuristic is to augment all arc flows to become zero, which is obviously a feasible solution (that is, we do not provide any flight service).

It should be noted that, due to Lagrangian relaxation, a rental arc cost is likely to be less than the modified cycle arc cost for the same station. Thus, if a lower bound solution contains rental airplanes while there are still available airplanes, then, obviously, a feasible solution can be improved by switching some of these rental arc flows to cycle arcs. This switch is repeated until all the available airplanes are used up or all rental arc flows are empty.

Solution process

The solution steps for the LRS are listed below:

Step 0: set $k = 0$, $\lambda^{k} = 2$, $\delta^{k} = 0$ and $\lambda^{k}_{s} = 0$ for all $s$.

Step 1: solve (3.2) optimally using the network simplex method to get a lower bound. If it is feasible and satisfies the complementary slackness, that is, $\sum_{s} u_{s} \left( \sum_{(i,j) \in H_{s}} X_{ij} - \delta_{s} \right) = 0$, then it is also an optimal solution. Stop the algorithm, otherwise, update the lower bound, $Z^{k}$.

Step 2: apply the Lagrangian heuristic to find an upper bound and update the upper bound, $Z^{k'}$.

Step 3: if the gap between the lower bound and the upper bound is within a specified tolerance, that is, $|(Z^{k'} - Z^{k})/Z^{k}| < \theta$, or the number of iterations reaches a limit, stop the algorithm.

Step 4: adjust $u$, using a sub-gradient method. In this research, we have tried the sub-gradient methods introduced by Fisher (1981) and Camerini et al. (1975) (called the CFM method). However, Fisher’s method did not perform well, with a maximum error gap of 37.5% in tests of all models presented in Section 4.
A decision support framework

(i.e. model "abc"). Although the CFM method performed better with a maximum error gap of 15.8%, it is not yet good enough. Therefore, we developed a modified sub-gradient method based on these two methods to improve the convergence. The modified method performed relatively well, with a maximum error gap of 11.3% (and with a maximum error gap of 3.8% in tests of most other models). The modifications are shown as follows:

\[ \hat{b}_i = \sum_{(i,j) \in E_i} X_{ij} - \delta_i, \quad \forall s \]

\[ b_k = \max\{0, -\frac{\hat{b}^k}{d^k} \} \]

\[ d^k = \hat{b}^k + b_k d^{k-1} \]

\[ r^k = \frac{\lambda^k (Z^u - Z(d^k))}{\sum_{(i,j) \in E_i} \sum_{\forall s} (X_{ij} - \delta_i)} \quad 0 < \lambda^k \leq 2 \]

(3.3)

\[ v^k = u^k + r^k d^k \]

\[ u_k^{k+1} = \max\{0, v^k\}, \quad \forall s, \]

where \( 0 < \lambda^* \leq 2 \) and \( \lambda^* \) will be divided by 1.2 (which is based on numerical experiments in the case study presented in Section 4), if the current lower bound is not updated for five iterations.

Step 5: set \( k = k + 1 \). Go to Step 1.

Since the solutions mentioned above do not show the route of each airplane, we suggest using a flow decomposition algorithm (Ahuja et al., 1993) to decompose the link flows into arc chains, each representing an airplane’s rotation. Although the arc chains may not be unique, the objective values for different patterns of arc chains are the same, because their fleet routes are the same. Carriers can choose several patterns and send them to the operating division for the application of aircraft maintenance and crew scheduling constraints. Thus, a satisfactory solution to both the planning and the operating divisions could be relatively easy to find. However, more practical path flow patterns can be looked into in the future, in order to meet aircraft maintenance regulations and crew scheduling.

4. CASE STUDY

The case study is based on data from a major Taiwan airline’s international operations from October to December in 1992 (China Airlines, 1993). There are 24 cities involved in its operations. The draft timetable is rotated once a week and includes 464 flights. About 80% of them (367 flights) are non-stop flights and 20% (97 flights) are one-stop flights. There are several types of aircraft involved in its operations, including B737s, Air Bus 300s, Air Bus 600s, MD11s, B74Ls, B744s and B747s. For simplification, we use three types of aircraft in this case study; in particular, type A indicates B737s (three airplanes) with 120 seats, type B includes Air Bus 300s, Air Bus 600s, MD11s and B74Ls (17 airplanes) with an average of 269 seats, and type C includes B744s and B747s (six airplanes) with an average of 403 seats. For the ease of testing, all the cost parameters are set according to the airline’s reports and Taiwan government regulation with reasonable simplifications (China Airlines, 1993; Civil Aeronautics Administration, 1984). In particular, every flight arc cost can be calculated according to the following formula: (flight arc cost = flight expense – flight revenue); the holding arc cost is set to be NT$65/h (about US$2.5/h) for type A aircraft, NT$130/h for type B aircraft, and NT$200/h for type C aircraft; the overnight cost is assumed to be included in the holding cost and is thus neglected, and the aircraft rental cost is set to be NT$665,064/week for type A aircraft, NT$4,409,054/week for type B aircraft, and NT$8,581,731/week.
for type C aircraft. Because the ownership cost of the carrier’s own airplanes is fixed in short-term operations, it is constant in terms of optimization. Therefore, the ownership cost is excluded from calculations of the flight cost, the holding cost, and the overnight cost. On the other hand, the aircraft rental cost should obviously include the ownership cost. As a result, the aircraft rental cost is much higher than the holding cost or the overnight cost.

C programs were coded for: (1) the analysis of raw data; (2) the building of the BMFM; (3) the development and solution of strategic models; and (4) the output of data. The case study was implemented on an HP735 workstation. Twenty-five scenarios were tested with problem sizes of up to 12,495 nodes and 26,670 arcs. Every strategy point is defined as indicated in Section 2.3. We consider the possible deletion of multi-stop flight segments (that is, strategy a) for every one-stop flight. For the simplification of strategy b, we only consider the adjustment of flight departure times within the carrier owned slot times. In particular, on the traffic line between Taipei and Tokyo we add three additional alternate flight arcs, respectively, before and after each draft flight arc. Furthermore, we consider possible aircraft rentals (strategy c) for every fleet.

As shown in Fig. 10, the algorithm developed for this research performs well for the basic model and most strategic models such as “a”, “b”, “c”, “ac” and “bc”. These models converge to within a 3.81% error in, at most, 35.77 min of CPU time. For example, Fig. 10 shows the convergence condition of the strategy “a” model. The computational performance for other models is similar and is not given here. As to the strategic models, “ab” and “abc”, their convergences are inferior and can only converge to within about 10% in more than 13 min, perhaps due to more complex side constraints. The algorithm would have to be improved to be satisfactorily applied to these two strategic models in actual operations.

The objective value of the strategic model “bc” is the best among all the strategic models, meaning that the routing is best for the systematic consideration of flight deletions, flight departure times and aircraft rentals. In other words, given 464 drafted flights and 26 airplanes, the carrier should delete 100 uneconomic flights (including 91 type B and nine type C flights), adjust the departure times of 21 flights (including three type B and 18 type A flights), rent nine airplanes (including five type B and four type C airplanes) and assign 11 type B flights to be served by the type C fleet, to achieve the most profitable timetable and fleet routes. As a result, 35 airplanes (including three type A, 22 type B and 10 type C airplanes) are scheduled to serve 364 flights (including 26 type A, 246 type B and 92 type C flights). On the other hand, the objective value of the basic model is the worst among all the strategic models, meaning that only considering flight deletions yields the least amount of profit. Note that, theoretically, the strategic model “abc” should give the best objective value; however, due to the performance of the developed LRS, its objective does not out-perform the other models. Therefore, an improved LRS may be developed to apply this strategy in the future. Finally, carriers can decide the best routing by comparing the results among all strategic models in addition to the objective values, and other criteria, such as the other real operation constraints, level of services, long-term policies, etc.

From column (10) in Table 1, it can be seen that all strategic models suggest the deletion of many type B and C draft flights. For example, the basic model suggests deleting 140 type B flights and 51 type C flights. The reasons are: (1) these two fleet sizes are not enough for

Fig. 10. The computational performance of the strategy “a” model.
Table 1. Results for all models

<table>
<thead>
<tr>
<th>Number of nodes</th>
<th>Number of arcs</th>
<th>Upper bound</th>
<th>Lower bound</th>
<th>Z_a</th>
<th>Z_b</th>
<th>Z_c</th>
<th>Number of constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategic model</td>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
</tr>
<tr>
<td>Basic model</td>
<td></td>
<td>85.69</td>
<td>-395.332.361</td>
<td>4033</td>
<td>0.259%</td>
<td>4033</td>
<td>4033</td>
</tr>
<tr>
<td>a</td>
<td></td>
<td>67.92</td>
<td>-410.290.857</td>
<td>4033</td>
<td>0.821%</td>
<td>4033</td>
<td>4033</td>
</tr>
<tr>
<td>b</td>
<td></td>
<td>125.5</td>
<td>-401.040.028</td>
<td>4033</td>
<td>0.753%</td>
<td>4033</td>
<td>4033</td>
</tr>
<tr>
<td>c</td>
<td></td>
<td>119.92</td>
<td>-415.824.716</td>
<td>4033</td>
<td>0.998%</td>
<td>4033</td>
<td>4033</td>
</tr>
<tr>
<td>abc</td>
<td></td>
<td>459.22</td>
<td>-428.030.084</td>
<td>4033</td>
<td>9.319%</td>
<td>4033</td>
<td>4033</td>
</tr>
<tr>
<td>Strategic model</td>
<td></td>
<td>2146.49</td>
<td>-442.665.941</td>
<td>4033</td>
<td>11.266%</td>
<td>4033</td>
<td>4033</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of deleted flights</th>
<th>Number of flights with deletions</th>
<th>Number of one-stop flights</th>
<th>Flights served by other fleet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategic model</td>
<td>(7)</td>
<td>(8)</td>
<td>(9)</td>
</tr>
<tr>
<td>Basic model</td>
<td>26</td>
<td>357 101</td>
<td>0 0</td>
</tr>
<tr>
<td>a</td>
<td>0 140 51</td>
<td>0 0 0</td>
<td>1 C</td>
</tr>
<tr>
<td>b</td>
<td>0 140 51</td>
<td>0 0 0</td>
<td>4 C</td>
</tr>
<tr>
<td>c</td>
<td>0 140 51</td>
<td>0 0 0</td>
<td>8 C</td>
</tr>
<tr>
<td>abc</td>
<td>0 140 51</td>
<td>0 0 0</td>
<td>11 C</td>
</tr>
<tr>
<td>Strategic model</td>
<td>26</td>
<td>337 101</td>
<td>0 0</td>
</tr>
<tr>
<td>a</td>
<td>0 140 51</td>
<td>0 0 0</td>
<td>1 C</td>
</tr>
<tr>
<td>b</td>
<td>0 140 51</td>
<td>0 0 0</td>
<td>4 C</td>
</tr>
<tr>
<td>c</td>
<td>0 140 51</td>
<td>0 0 0</td>
<td>8 C</td>
</tr>
<tr>
<td>abc</td>
<td>0 140 51</td>
<td>0 0 0</td>
<td>11 C</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of side contracts</th>
<th>Number of side constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategic model</td>
<td>(10)</td>
</tr>
<tr>
<td>Basic model</td>
<td>26</td>
</tr>
<tr>
<td>a</td>
<td>0 140 51</td>
</tr>
<tr>
<td>b</td>
<td>0 140 51</td>
</tr>
<tr>
<td>c</td>
<td>0 140 51</td>
</tr>
<tr>
<td>abc</td>
<td>0 140 51</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of draft flights</th>
<th>Number of draft flights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategic model</td>
<td>(11)</td>
</tr>
<tr>
<td>Basic model</td>
<td>26</td>
</tr>
<tr>
<td>a</td>
<td>0 140 51</td>
</tr>
<tr>
<td>b</td>
<td>0 140 51</td>
</tr>
<tr>
<td>c</td>
<td>0 140 51</td>
</tr>
<tr>
<td>abc</td>
<td>0 140 51</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of aircraft rentals</th>
<th>Number of aircraft rentals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategic model</td>
<td>(12)</td>
</tr>
<tr>
<td>Basic model</td>
<td>26</td>
</tr>
<tr>
<td>a</td>
<td>0 140 51</td>
</tr>
<tr>
<td>b</td>
<td>0 140 51</td>
</tr>
<tr>
<td>c</td>
<td>0 140 51</td>
</tr>
<tr>
<td>abc</td>
<td>0 140 51</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of one-stop segments</th>
<th>Number of one-stop segments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategic model</td>
<td>(13)</td>
</tr>
<tr>
<td>Basic model</td>
<td>26</td>
</tr>
<tr>
<td>a</td>
<td>0 140 51</td>
</tr>
<tr>
<td>b</td>
<td>0 140 51</td>
</tr>
<tr>
<td>c</td>
<td>0 140 51</td>
</tr>
<tr>
<td>abc</td>
<td>0 140 51</td>
</tr>
</tbody>
</table>
the draft timetable which is planned for high demand conditions; (2) the drafted flights (scheduled by hand) are not well connected in terms of network operations; and (3) a significant number of drafted flights are not economic (not profitable), from the system perspective. In addition, the strategic models are effective, as shown in the results. In particular, concerning aircraft rentals, the number of flights deleted is apparently decreased. For example, looking at column (13) for strategy "c", the model suggests renting five type B airplanes and five type C airplanes; thus a total of 107 draft flights is deleted, which is obviously less than the basic model (191 flights), model "a" (190 flights) or the model "b" (187 flights). Looking at column (11), a one-stop flight in model "ac" and two one-stop flights in model "abc" indicate that some one-stop flight segments should be deleted to improve system profit. Besides, from column (12) for the model containing "b", the continuity of flights in the timetable drafts can apparently be improved. For example, departure times for 11 flights in model "b" and model "ab", 21 flights in model "bc" and 7 flights in model "abc" should be adjusted. Finally, some larger aircraft can be used to serve the smaller type draft flights to efficiently utilize all of the fleets. For instance, from column (14), several type B draft flights in each scenario can be served by the type C fleet so as to improve system profits.

The flow decomposition method was applied to trace the path of each airplane. In particular, fleet flows are decomposed into several arc chains, each representing an airplane's rotation. For example, the results of the basic model show that 26 type A drafted flights are served by three type A airplanes. Therefore, given the type A fleet network flows, we apply the flow decomposition method to trace three arc chains for these three airplanes. As a result, we find three aircraft routes to serve these 26 flights. An example of an airplane path (with block times for flights) for each type of fleet given by the algorithm, is shown in Fig. 11. Other aircraft routes can be traced similarly and are not shown. Note that the arc

![Fig. 11. An example of airplane routes.](image-url)
chains may not be unique. When applying the models in practice, carriers may solve several arc chain patterns, then send them to the operating division for the application of aircraft maintenance and crew scheduling constraints. Hence, there will be more choices for the operating division to choose the best routing. Even if all the patterns are infeasible, then instead of revising only one schedule as is done currently, several alternate patterns with the same best profit can provide more flexibility for schedule revisions. Thus, both the scheduling process and the quality of a schedule could be improved.

Because the algorithm is a problem-oriented heuristic, its performance could be affected by problem parameters. To further understand the algorithm performance, we tested 17 other scenarios of the same type of network flow problems. These 17 scenarios were created by changing parameters such as the available fleet size (scenarios 1–5), the aircraft rental cost (scenarios 6–13), and the draft schedule based on the draft timetable used above (scenarios 14–17). The results are summarized in Table 2 and show that the algorithm performs well for these scenarios. All of them converged to within a 1% error in, at most, 3 min.

From the results in Tables 1 and 2, we found that, if there were not many side constraints, typically for simpler strategic models, the convergence of the algorithm was good. Otherwise, the convergence could be very slow, and the error gap could be significant. Knowing this, carriers should be more careful when applying the algorithm to solve complicated models (for example, models "ab" and "abc” in the case study) than to solve other strategic models. The reason for such problems might be guessed as follows: when the number of side constraints is increased too much, the Lagrangian heuristic might lose the upper bound due to a significant adjustment (instead of perturbation) on the lower bound solution, or the Lagrangian sub-problem might lose the lower bound due to the inherent duality gap. For the former reason, it might be helpful to reduce the error gap by developing another Lagrangian heuristic to find a better upper bound, for example, using more effective rules for flow augmentation. For the latter, it might be useful to develop another approach to find a better lower bound, for example, relaxing flow conservation constraints or using LP relaxation. If, unfortunately, after the improvement there is a significant duality (relaxation) gap, then some form of enumeration procedure, such as branch and bound, using the Lagrangian lower bound to help reduce the amount of concentration required could be developed. After all, the improvements of the algorithm could be directions for future research. Because the test examples are limited in the case study which is only for demonstration, more tests for various strategic models with different problem sizes should be conducted for carriers to understand the limitations before applying these strategic models to actual operations.

5. CONCLUSIONS

Motivated by the inefficient and ineffective fleet routing and flight scheduling for Taiwan airline carriers and the lack of multi-fleet routing and flight scheduling models in practice, this research develops a framework to help carriers adjust draft timetables and multi-fleet routes when market demand conditions are expected to change in the near future. The framework is based on a multi-fleet time-space network representing a basic model from which several strategic models are developed to help carriers in fleet routing and flight scheduling. These models are formulated as integer multiple commodity network flow problems. Lagrangian relaxation, accompanied by the network simplex method, a self-developed Lagrangian heuristic, and a modified sub-gradient method are developed to solve the problems. A flow decomposition algorithm is also suggested in order to trace every aircraft route.

To show how to apply this framework in the real world, a case study regarding the international operations of a major Taiwan airline was performed. Twenty-five scenarios were tested with substantial problem sizes of up to 12,495 nodes and 26,670 arcs. The LRS developed in this research performed well for most test problems. The LRS requires improvement to be applied to the other two complicated models in actual operations. Although the results show that the framework could be useful for the tested service network, more tests should be conducted for carriers to understand the limitations before applying the strategic
Table 2. Test results for algorithm performance

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Computation time (s)</th>
<th>Lower bound $Z_L$</th>
<th>Upper bound $Z_U$</th>
<th>$(Z_U - Z_L)/Z_L$</th>
<th>Number of nodes</th>
<th>Number of arcs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>1</td>
<td>89.44</td>
<td>382,849,300</td>
<td>379,301,254</td>
<td>0.935%</td>
<td>4033</td>
<td>4033</td>
</tr>
<tr>
<td>2</td>
<td>82.18</td>
<td>-388,852,768</td>
<td>-387,663,134</td>
<td>0.307%</td>
<td>4033</td>
<td>4033</td>
</tr>
<tr>
<td>3</td>
<td>111.43</td>
<td>-400,905,918</td>
<td>-400,328,344</td>
<td>0.144%</td>
<td>4033</td>
<td>4033</td>
</tr>
<tr>
<td>4</td>
<td>79.89</td>
<td>-401,476,236</td>
<td>-405,490,523</td>
<td>0.490%</td>
<td>4033</td>
<td>4033</td>
</tr>
<tr>
<td>5</td>
<td>93.64</td>
<td>-412,237,657</td>
<td>-410,603,702</td>
<td>0.398%</td>
<td>4033</td>
<td>4033</td>
</tr>
<tr>
<td>6</td>
<td>124.87</td>
<td>-450,321,218</td>
<td>-447,157,468</td>
<td>0.708%</td>
<td>4033</td>
<td>4033</td>
</tr>
<tr>
<td>7</td>
<td>102.23</td>
<td>-442,163,957</td>
<td>-437,866,309</td>
<td>0.981%</td>
<td>4033</td>
<td>4033</td>
</tr>
<tr>
<td>8</td>
<td>112.98</td>
<td>-432,767,725</td>
<td>-426,628,797</td>
<td>0.966%</td>
<td>4033</td>
<td>4033</td>
</tr>
<tr>
<td>9</td>
<td>145.72</td>
<td>-423,676,679</td>
<td>-421,067,055</td>
<td>0.620%</td>
<td>4033</td>
<td>4033</td>
</tr>
<tr>
<td>10</td>
<td>128.18</td>
<td>-409,825,70</td>
<td>-405,482,056</td>
<td>0.987%</td>
<td>4033</td>
<td>4033</td>
</tr>
<tr>
<td>11</td>
<td>121.1</td>
<td>-403,718,114</td>
<td>-401,164,343</td>
<td>0.637%</td>
<td>4033</td>
<td>4033</td>
</tr>
<tr>
<td>12</td>
<td>163.47</td>
<td>-400,308,773</td>
<td>-397,092,498</td>
<td>0.810%</td>
<td>4033</td>
<td>4033</td>
</tr>
<tr>
<td>13</td>
<td>115.26</td>
<td>-397,933,119</td>
<td>-394,330,339</td>
<td>0.914%</td>
<td>4033</td>
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<tr>
<td>14</td>
<td>127.24</td>
<td>-400,354,834</td>
<td>-397,326,646</td>
<td>0.762%</td>
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<tr>
<td>15</td>
<td>119.2</td>
<td>-401,364,513</td>
<td>-397,886,391</td>
<td>0.874%</td>
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<tr>
<td>16</td>
<td>152.46</td>
<td>-400,947,460</td>
<td>-398,419,208</td>
<td>0.635%</td>
<td>4033</td>
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</tr>
<tr>
<td>17</td>
<td>165.49</td>
<td>-404,422,110</td>
<td>-401,654,804</td>
<td>0.689%</td>
<td>4033</td>
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</tr>
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</table>
models to actual operations. Because the case study is only for demonstration at the current stage, the evaluation of the application of this framework to actual operations is left to future work.

In contrast to Simpson’s, Lee’s or the past single-fleet routing models, this research has at least the following contributions:

(1) It developed a decision support framework to improve the traditional scheduling process (especially in Taiwan).

(2) It developed various multi-fleet routing models which are useful for Taiwan airline carriers with multiple types of fleets in operation. As long as the basic model is created, the strategic models can be easily modified from the basic model and can be used by carriers to evaluate their draft timetables and fleet routes.

(3) Besides the modelling of cross cycle flights, we proposed here several practical routing strategies, such as aircraft rental, slot rental, slot negotiation, and the evaluation of a one-stop flight with three non-stop flights. These strategies are incorporated into the modelling as well.

(4) We successfully applied the technique of flow augmentation to develop a Lagrangian heuristic which perturbs the optimal solution of the Lagrangian sub-problem, thus finding a good upper bound.

(5) We developed a modified sub-gradient method to adjust good Lagrangian multipliers by combining and modifying Fisher’s method (1981) and the CFM method. This modified method was tested in our case study and was shown to be better than Fisher’s and the CFM methods.

(6) We performed a case study with substantial problem sizes in the context of multi-fleet operations. Most models in the framework were shown to be good and could be useful in actual operations.

In this research, a multi-stop flight is evaluated so as to delete some of its segments. How to combine the flight segments of different flights to form a multi-stop flight is not considered here and can be another direction for future research. Besides, it can also be a topic of future research for how to combine several smaller draft flights to form a larger type of flight. Finally, the incorporation of other routing constraints, for example, maintenance and crew scheduling, or different objectives involved in actual operations, can be further directions of future research.

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REFERENCES


